Practical SAT Solving (ST 2024) | Assignment 1

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Algorithm Engineering (KIT) 2025-05-06 – 2024-05-20

1 Competition: Graph Coloring (5(+7) Points)

Implement a SAT-based graph vertex coloring solver. Your application should take as a single command line argument a DIMACS file with a graph. It needs to find the smallest number of colors needed to color the graph such that no adjacent vertices share the same color. The application should output the number of colors required. The fastest solver gets seven bonus points.

Code skeleton: `https://github.com/satlecture/kit2025/blob/main/code/src/graphcol.cc`
Benchmark instances: `https://github.com/satlecture/kit2024/tree/main/exercises/coloring`

2 Pythagorean Triples (3+3 Points)

(a) Find a coloring for the numbers $1 \leq i \leq 1000$ such that no Pythagorean triple is monochromatic.
(b) Estimate the number of variables and clauses in the Pythagorean triples encoding from the lecture (as a function of n).

3 Recognizing Hidden Horn (6 Points)

Let F be an arbitrary CNF formula. Let R_F be a 2-SAT formula that contains the clause $\{l_1, l_2\}$ iff there is a clause $C \in F$ such that $\{l_1, l_2\} \subseteq C$. Prove that if R_F is satisfiable, then F is a Hidden Horn formula.

4 Tseitin Encoding (3+3 Points)

Encode the following formula into CNF using (a) the Tseitin encoding and (b) the Plaisted-Greenbaum encoding.

$$(\overline{x_1} \land \overline{(x_3 \Leftrightarrow x_2)}) \lor ((x_3 \to \overline{x_4}) \land (x_1 \to (x_2 \land \overline{x_3})) \land (x_4))$$

5 Resolution (3 Points)

Construct a resolution refutation (i.e., proof of unsatisfiability) for the following formula.

$$\big\{\{x_3, x_4, \overline{x}_1, x_5\}, \{\overline{x}_3, x_4, x_5\}, \{x_3, \overline{x}_4, \overline{x}_1\}, \{x_1, x_2\}, \{x_1, \overline{x}_2\}, \{\overline{x}_1, \overline{x}_5\}, \{\overline{x}_3, \overline{x}_4, x_5\}\big\}$$