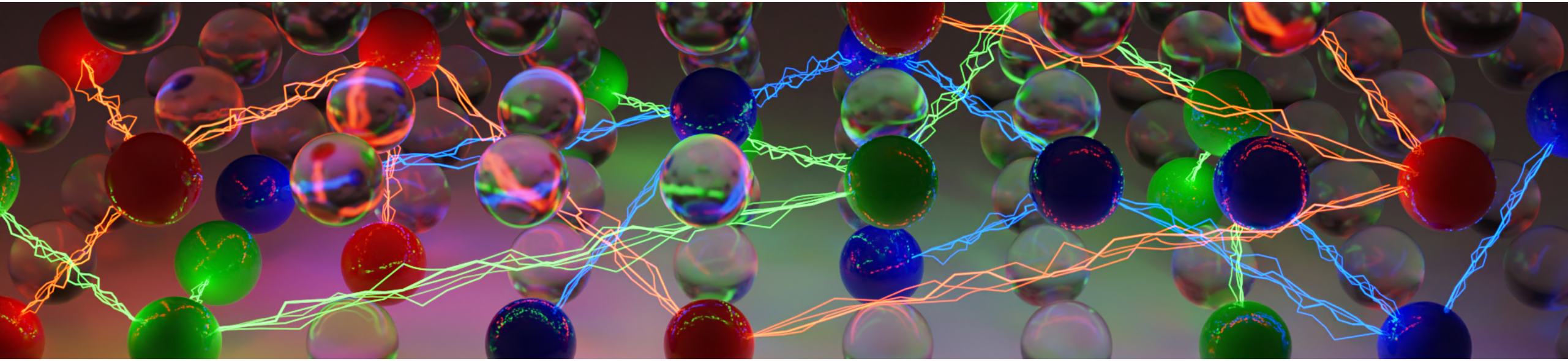
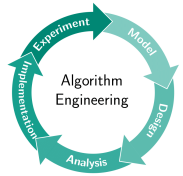


Practical SAT Solving



Lecture 8 – Parallel SAT Solving

Markus Iser, Dominik Schreiber | June 16, 2025

SAtRes
Scalable Automated Reasoning

Overview

Recap Lecture 7

- Propagation-based Redundancy notions
- Proof systems: Resolution, Extended Resolution, Blocked Clauses, Implication-based Redundancy
- Autarkies, Conditional Autarkies, and Satisfaction Driven Clause Learning (SDCL)

Today: Parallel SAT Solving

Parallel SAT solving approaches

- Basic search space splitting, Clause sharing, Cube&Conquer, Portfolio solvers

A deep dive into Mallob

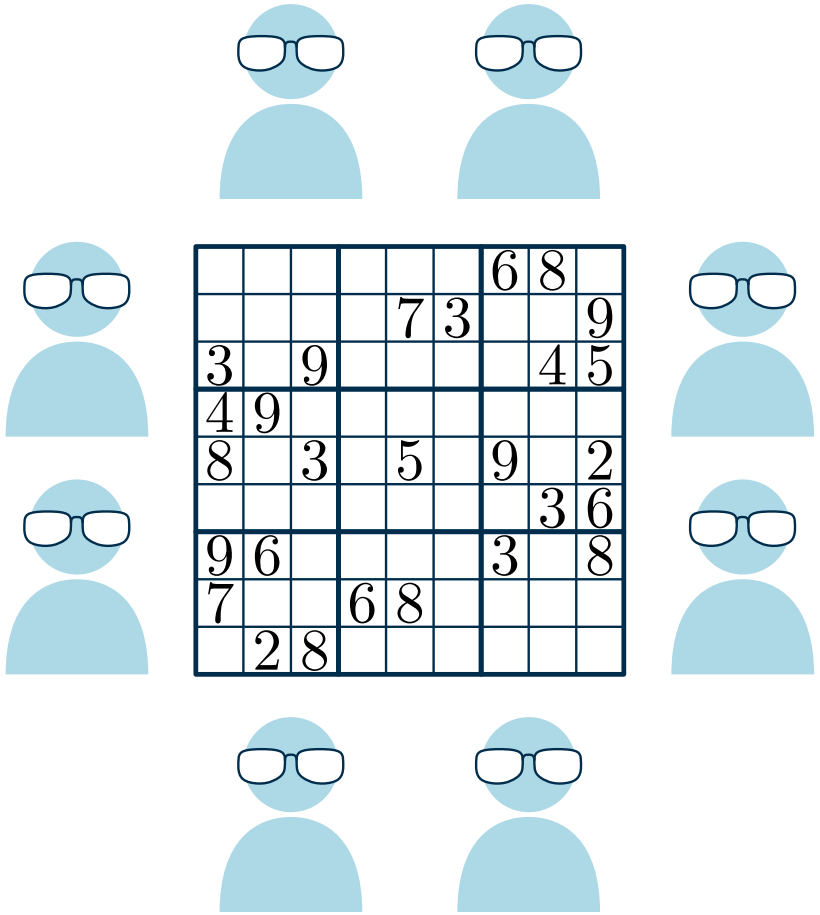
- Overview, Scalable clause sharing, Experiments and insights

Parallel Solving: An analogy

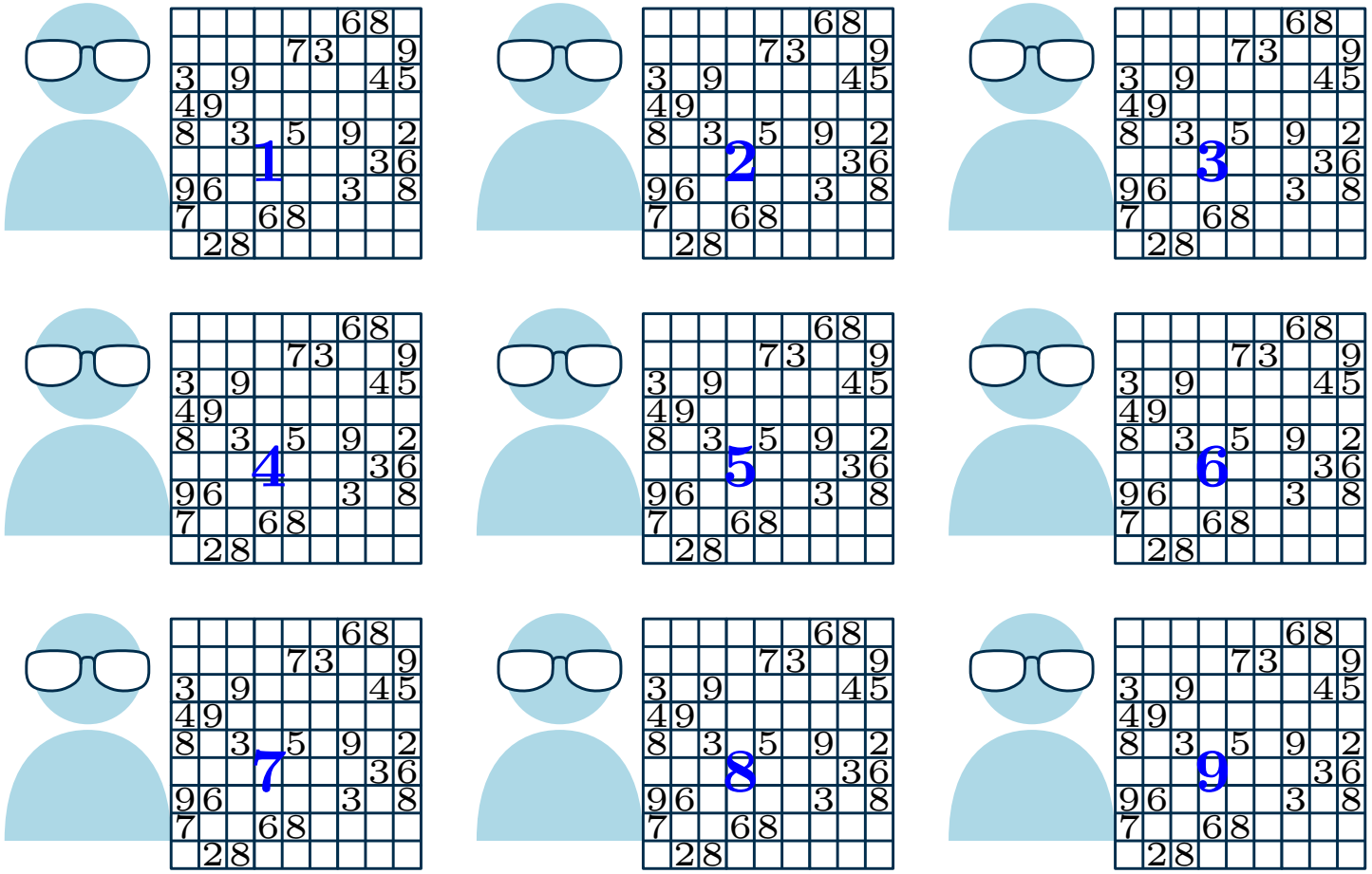
The Assembly of Nerds [23]

- Complex and large logic puzzle
- *n* puzzle experts at your disposition
  — anti-social: work best if left undisturbed

How do we employ and “orchestrate” our experts?



Approach I: Search Space Partitioning



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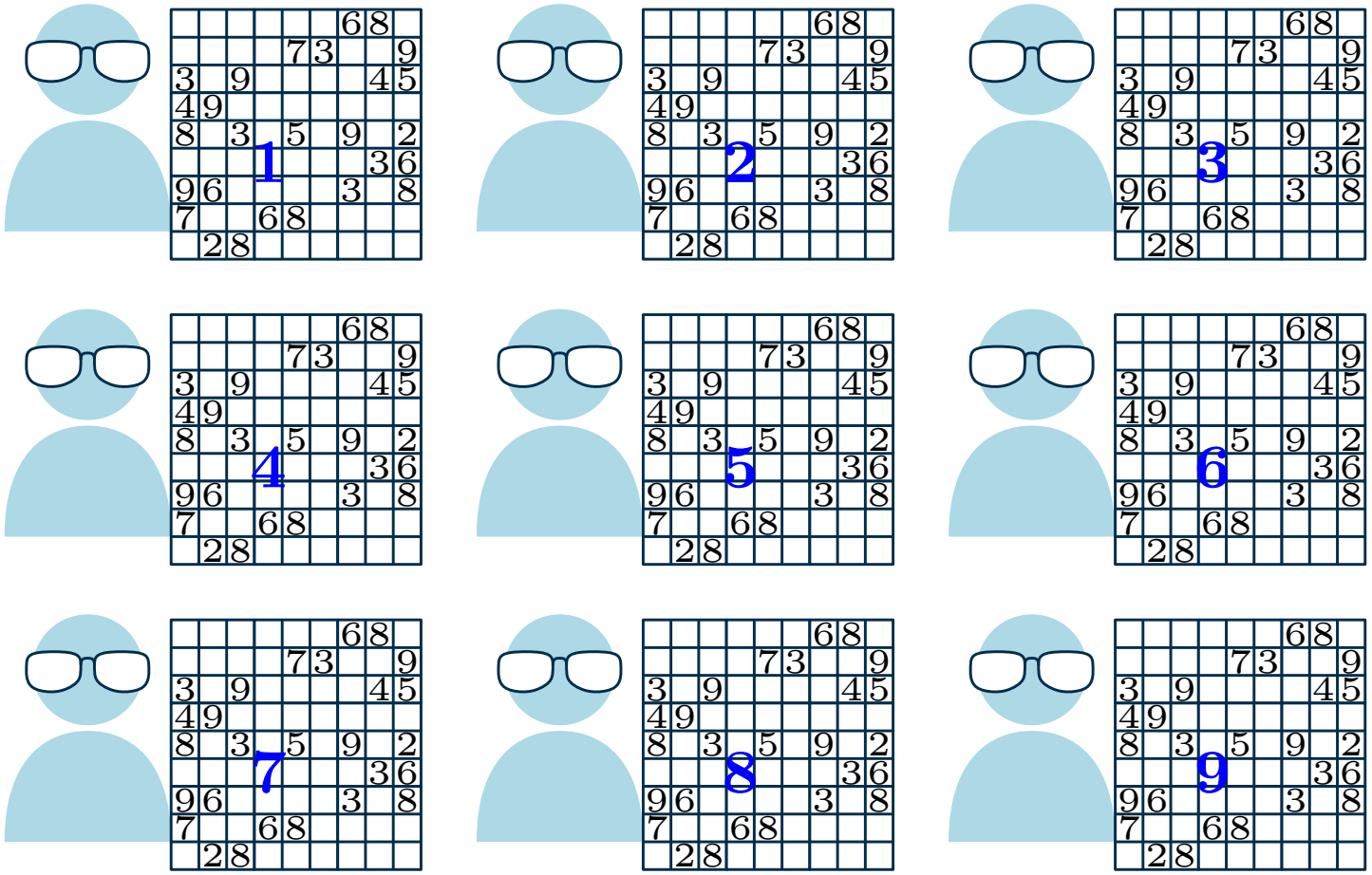


- Partition search space at some decisions
  $\Rightarrow$ Independent subproblems

Explicit Partitioning

Böhm & Speckenmeyer (1994-1996) [5]: 1st Parallel DPLL Implementation

Explicit Load Balancing

- Completely distributed (no leader / worker roles)
- A list of partial assignments is generated
- Each process receives the entire formula and a few partial assignments
- Each process can be worker or balancer:
  - Worker: solve or split the formula, use the partial assignments
  - Balancer: estimate workload, communicate, stop
- Switch to balancer whenever worker is finished

Explicit Partitioning

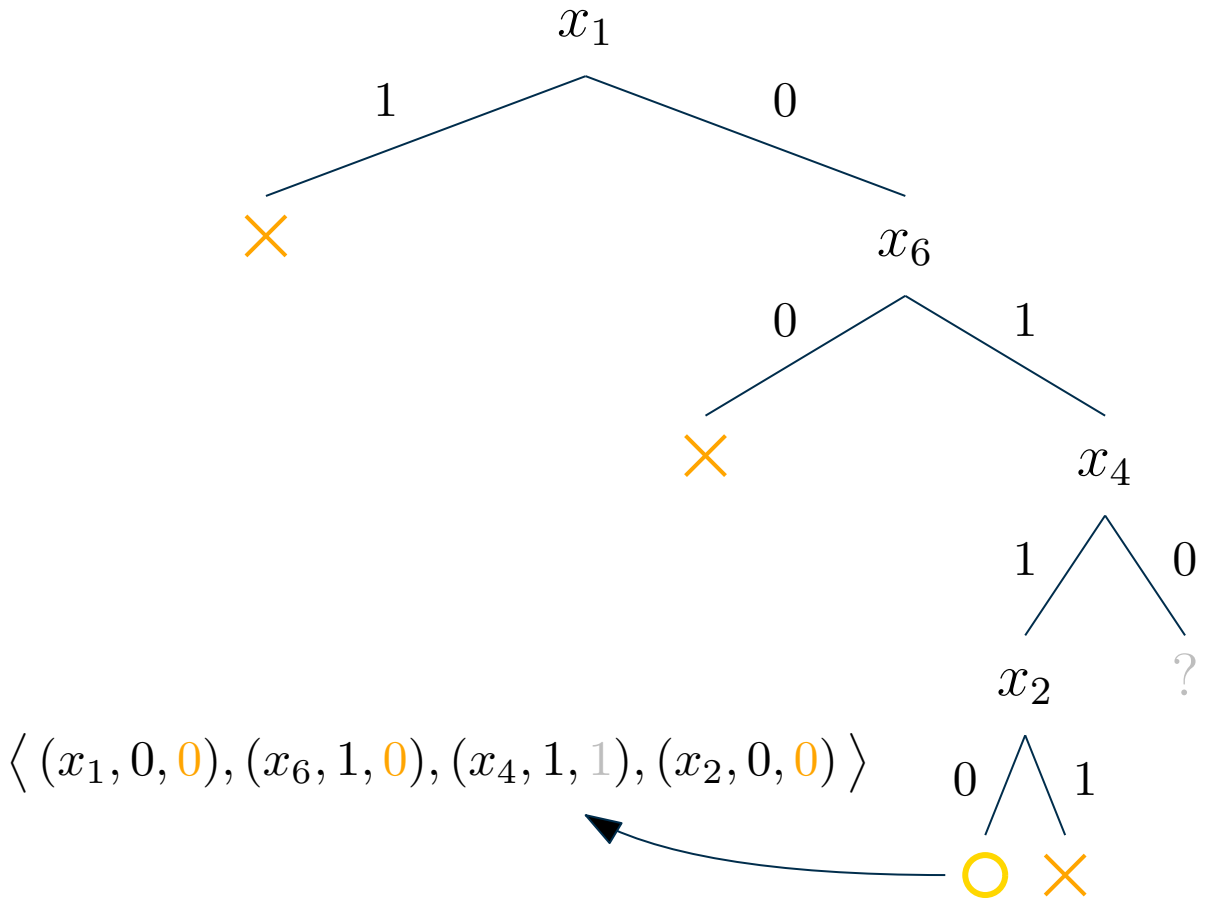
"**PSATO**: a Distributed Propositional Prover and its Application to Quasigroup Problems", Zhang et al. (1996) [26]

Centralized leader-worker architecture

- Communication only between leader and workers
- Leader assigns partial assignments using Guiding Path
  - Each node in the search tree is open or closed
    — closed = branch is explored / proven unsat
  - Leader splits open nodes and assigns job to workers
- Workers return Guiding Path when terminated by leader
- Modern features of fault tolerance, preemption of solving tasks

Explicit Partitioning

Guiding Path: List of triples (variable, branch, open)



Explicit Partitioning Solvers

SATZ (Jurkowiak et al., 2001) [15]: *Work stealing* for workload balancing

- An idle worker requests work from the leader
- The leader splits the work of the most loaded worker
- The idle worker and most loaded worker get the parts

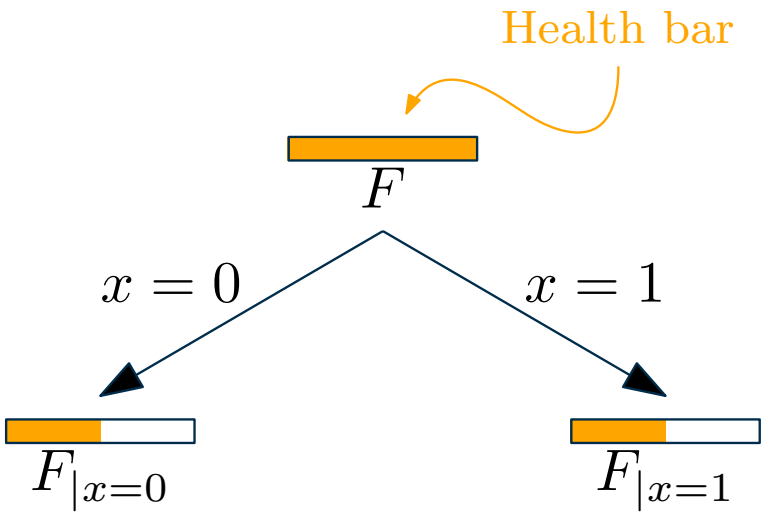
PaSAT (Blochinger et al., 2001-2003) [4]

- First parallel CDCL with clause sharing
- Similar to PSATO/SATZ: leader/worker, guiding path, work stealing

ySAT (Feldman et al., 2004) [8]

- First shared-memory parallel solver
- Multi-core processors started to be popular
- uses same techniques as the previous solvers (guiding path etc.)

Problems with Partitioning [25]



What we want: **Even splits**

- Split yields sub-formulas of similar difficulty
- Balanced partitioning of work
- Few or no dynamic (re-)balancing needed

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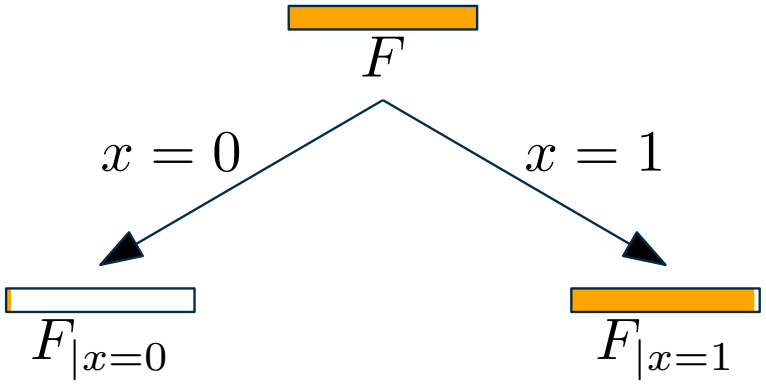

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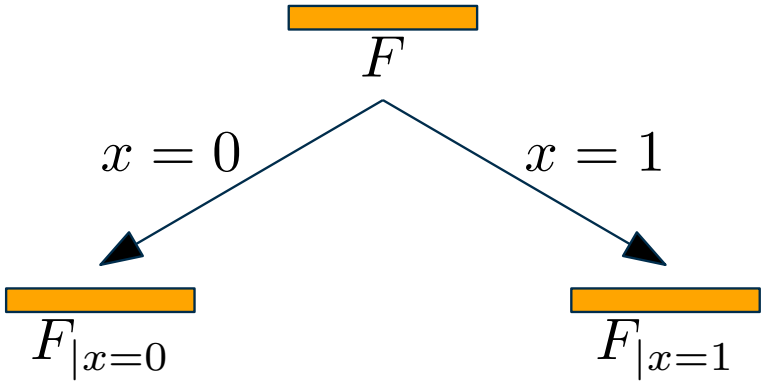

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Bogus splits

- Both $F_{|x=0}$ and $F_{|x=1}$ are just as hard as F
- Divide&Conquer becomes Multiply&Surrender!

Cube and Conquer

The Cube&Conquer paradigm (Heule & Biere, 2011) [14]

Generate a large amount (millions) of partial assignments (“**cubes**”) and randomly assign them to workers.

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- Partial assignments are generated using a look-ahead solver
  (breadth-first search up to a limited depth)
- Best performance mostly with problem-specific decision heuristics
- State-of-the-art for hard combinatorial problems
  - Used to solve the “Pythagorean Triples” problem ($\sim$200TB proof) [13]
  - ... or more recently “Schur Number 5” ($\sim$2PB proof) [12]
- Examples: March (Heule) + iLingeling (Biere) introduced in 2011; Treengeling (Biere) [2]

Parallel Portfolios: An analogy

The Assembly of Nerds

- Complex and large logic puzzle
- *n* puzzle experts at your disposition
  — individual mindsets, approaches, strengths & weaknesses
  — anti-social: work best if left undisturbed

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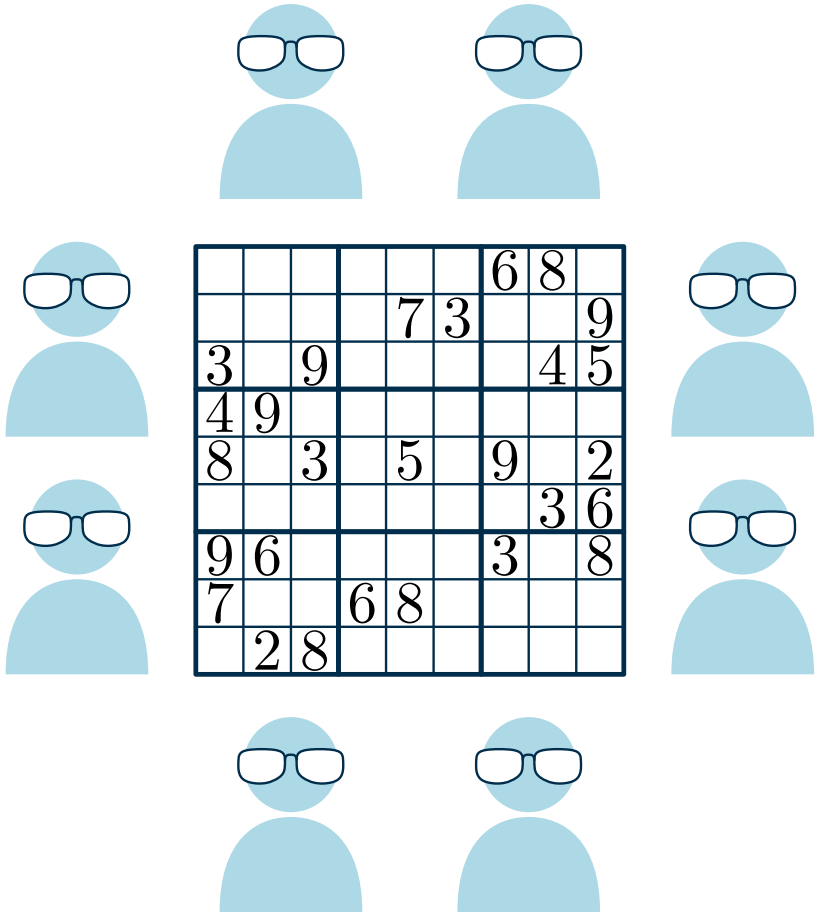
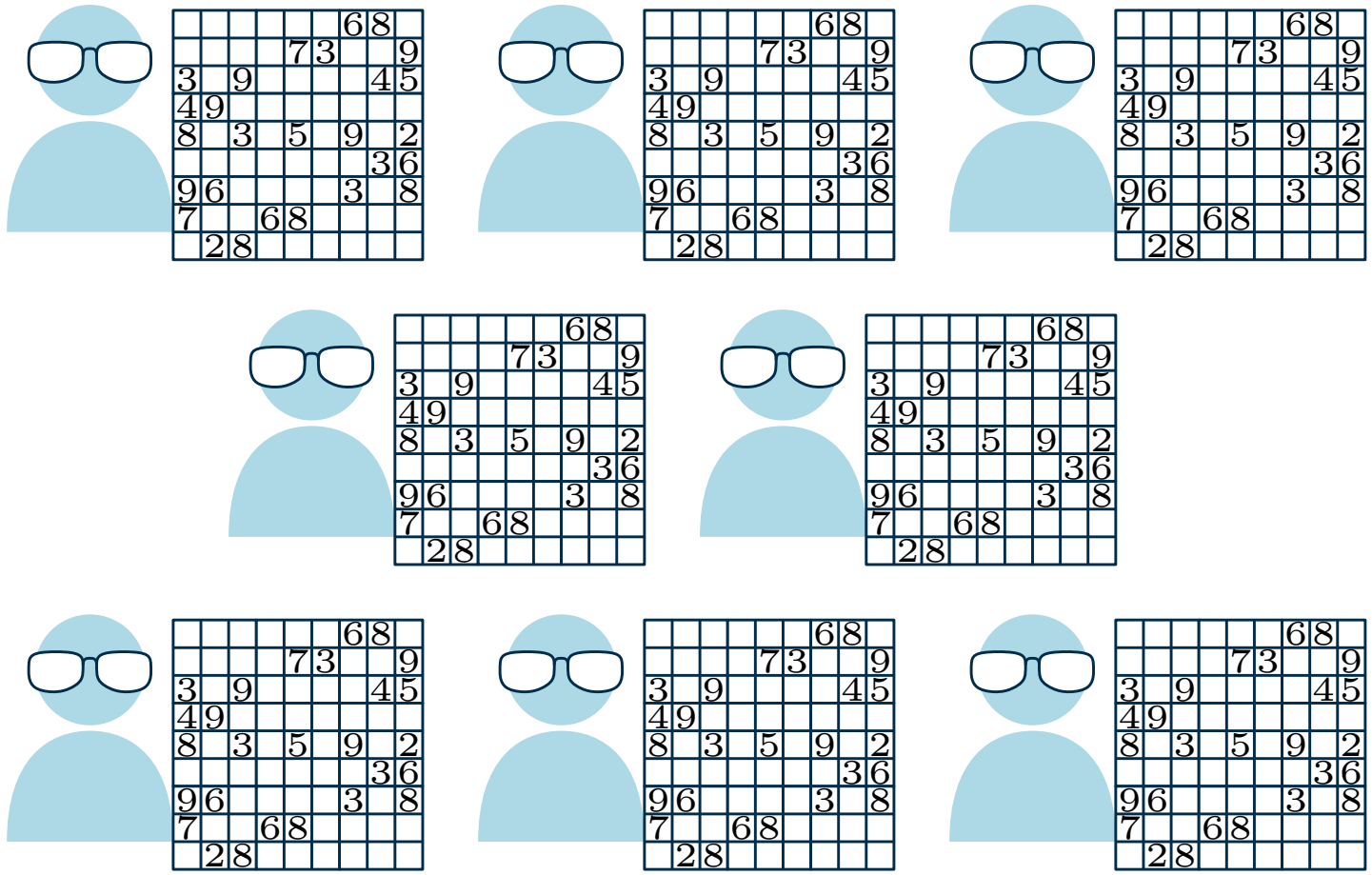


Approach II: Pure Portfolio



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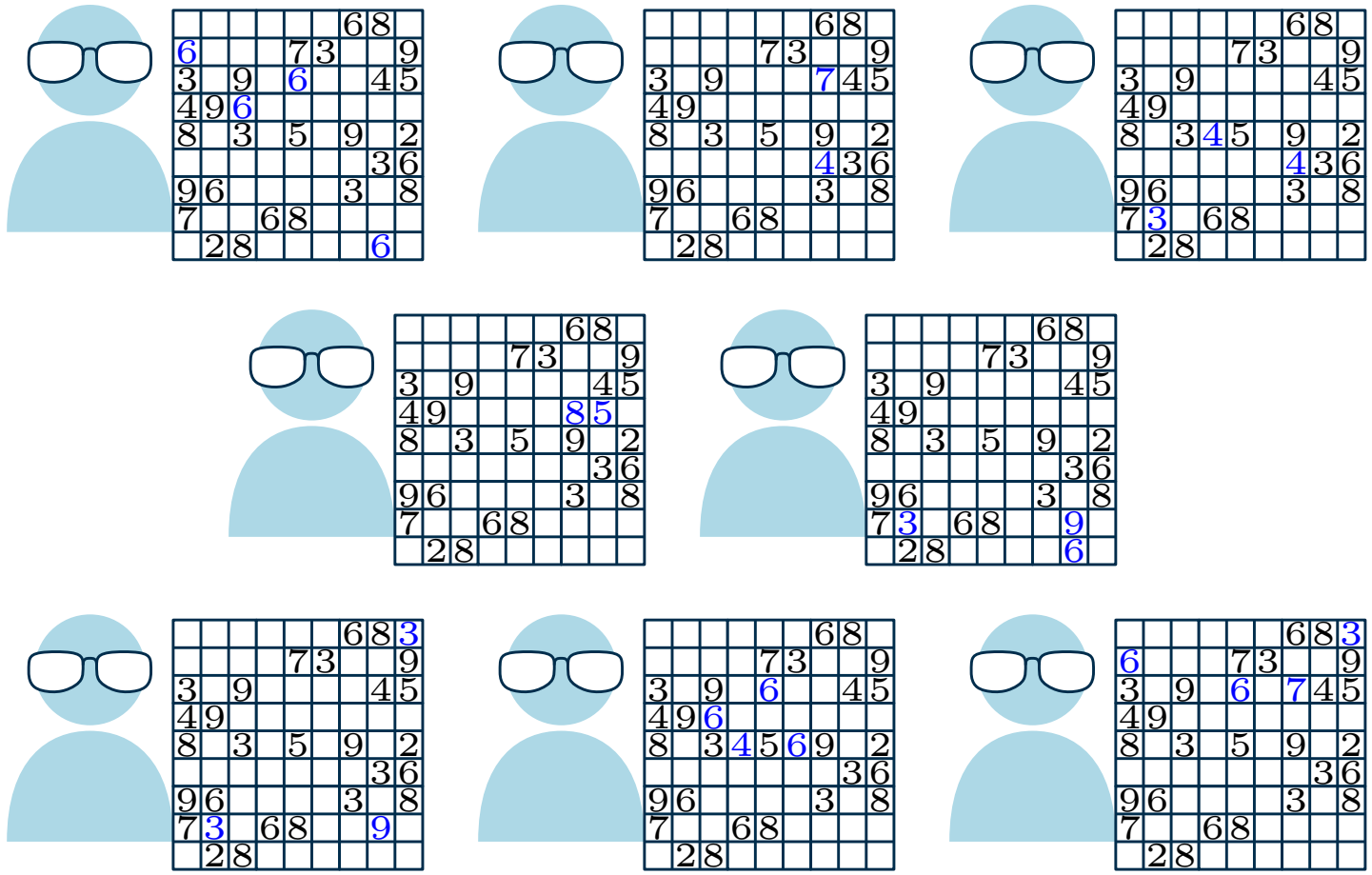

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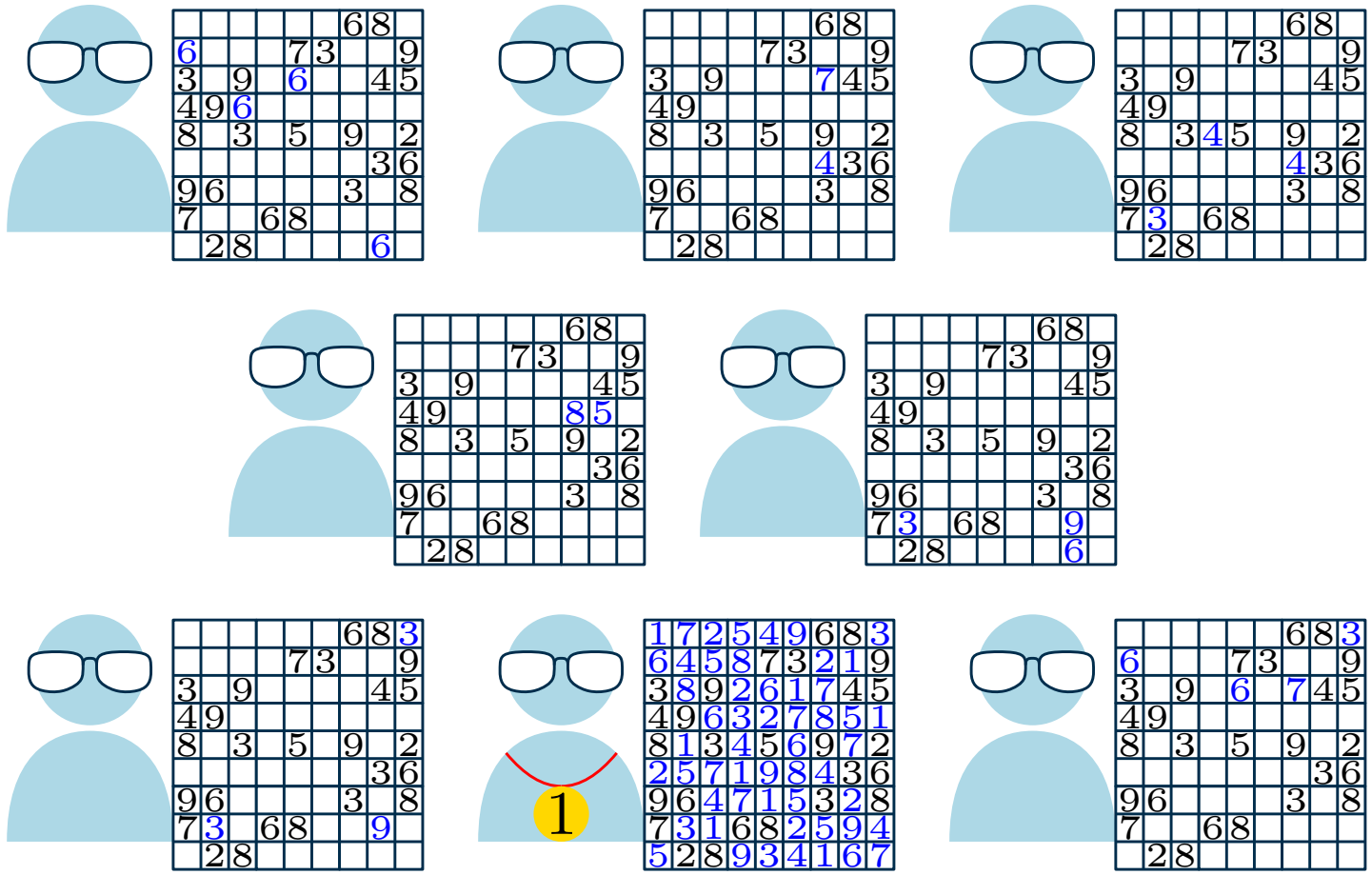

Pure Portfolio: Oracle view vs. Speedup view

Virtual Best Solver (VBS) / Oracle

Consider n algorithms $A_1, \ldots, A_n$ where for each input x, algorithm A_i has run time $T_{A_i}(x)$.
The Virtual Best Solver (VBS) for $A_1, \ldots, A_n$ has run time $T^*(x) = \min\{T_{A_1}(x), \ldots, T_{A_n}(x)\}$.

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Parallel speedup [20]

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where Q is the best available sequential algorithm.

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Pessimist: A pure portfolio never achieves actual speedups!

- There is always a sequential algorithm performing at least as well
- Consequence: Not resource efficient, not scalable

Pure SAT Portfolios

ppfolio: Winner of Parallel Track in the 2011 SAT Competition [18]

- Just a bash script combining the best sequential solvers from 2010:
  ```
  ~$ ./solver1 f.cnf & ./solver2 f.cnf & ./solver3 f.cnf & ./solver4 f.cnf
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- Bits by O. Roussel, the author of `ppfolio`:
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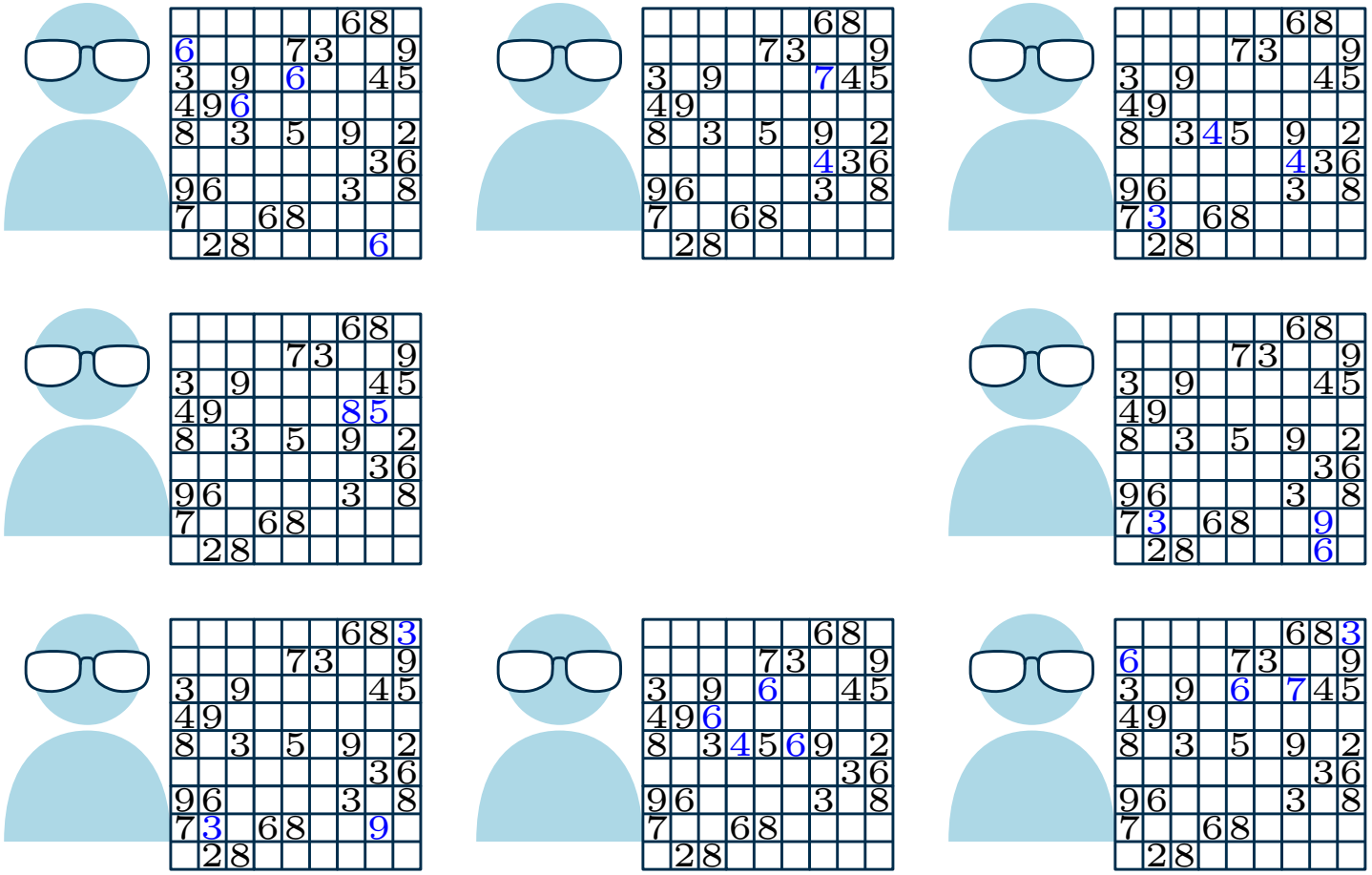
- Rationale: Different solvers are designed differently, excel on different instances
  — hope of orthogonal search behavior
- Pure portfolios no longer permitted in SAT Competitions

Approach II+: Cooperative Portfolio



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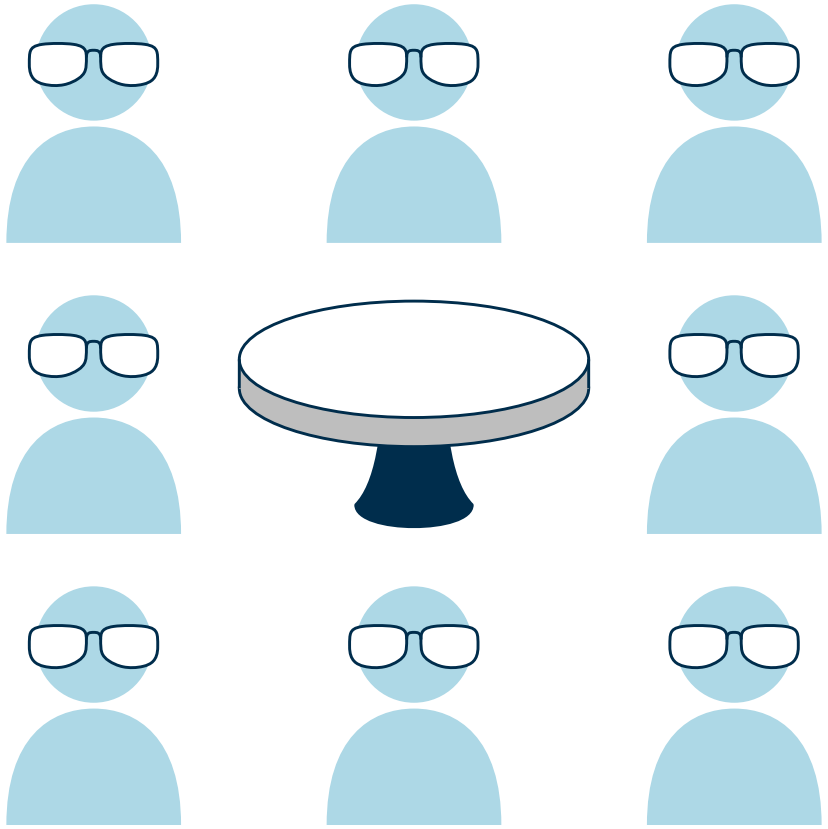

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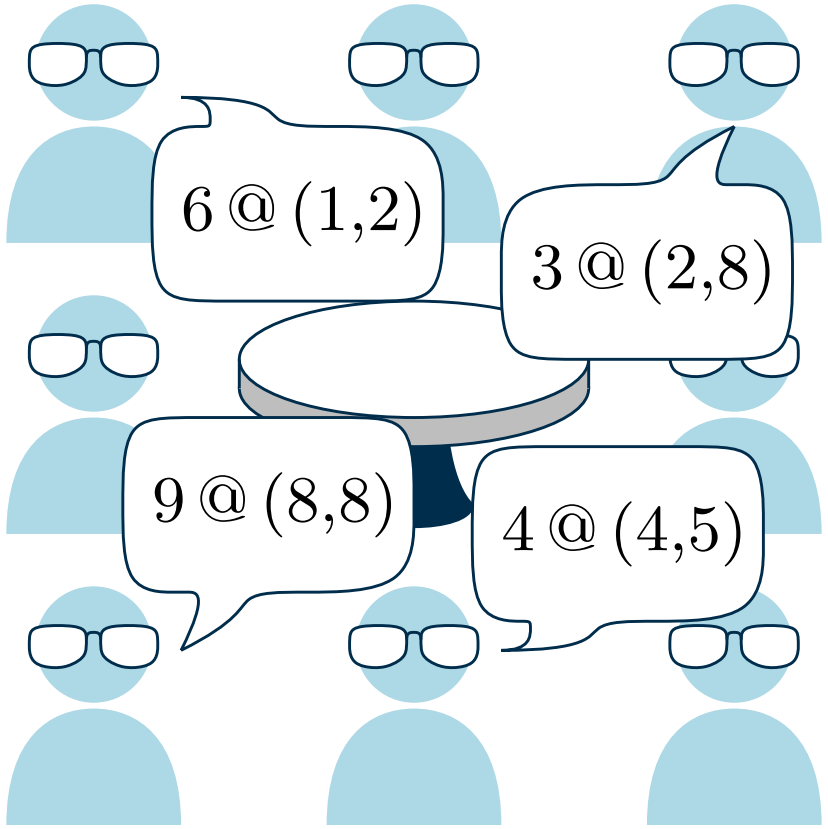

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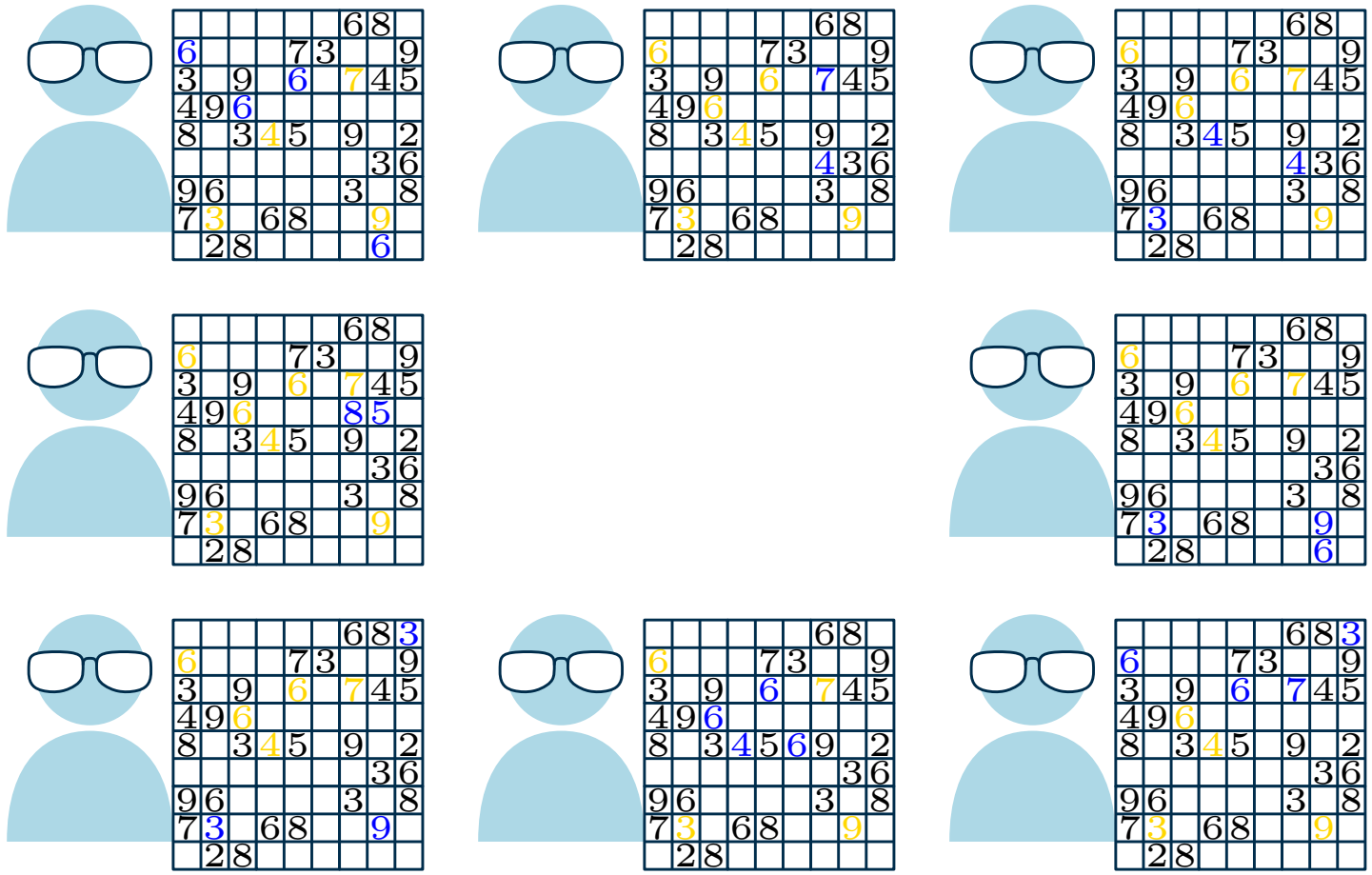

Cooperative Portfolio

Assembly of Nerds, enhanced

- The experts periodically gather for brief standup meetings
- Via some protocol, the experts exchange the most valuable insights gained since the last meeting
- Solving continues — each expert may use the shared insights at their own discretion

Equivalent to “insights” in SAT solving:

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Equivalent to “insights” in SAT solving: **learnt (conflict) clauses**

- Explored branch of search space — safe to prune
- Potential step for deriving unsatisfiability
- Result: Parallel search space pruning procedure

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Combination of portfolio idea + clause exchanges: **Clause sharing portfolio solvers**

Clause Sharing Portfolios: Design Space

Portfolio considerations

- Which sequential solvers to employ?
- How to diversify solvers?
  — different search algorithms, selection heuristics, restart intervals, . . .
  — different random seeds, initial phases, input permutations, . . .

```
void Cadical::diversify(int seed) {
    solver->set(name: "seed", val: seed);
    switch (getDiversificationIndex() % getNumOriginalDiversifications()) {
    case 0: okay = solver->set(name: "phase", val: 0); break;
    case 1: okay = solver->configure("sat"); break;
    case 2: okay = solver->set(name: "elim", val: 0); break;
    case 3: okay = solver->configure("unsat"); break;
    case 4: okay = solver->set(name: "condition", val: 1); break;
    case 5: okay = solver->set(name: "walk", val: 0); break;
    case 6: okay = solver->set(name: "restartint", val: 100); break;
    case 7: okay = solver->set(name: "cover", val: 1); break;
    case 8: okay = solver->set(name: "shuffle", val: 1) && solver->set(name: "
    case 9: okay = solver->set(name: "inprocessing", val: 0); break;
```

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Clause exchange considerations

- How often to share? (immediate/eager? delayed/lazy? periodic?)
- How many clauses to share? (fixed volume? fixed quality criteria?)
- Which clauses to share? (shortest? lowest LBD?)
- How to implement sharing? (all-to-all? leader-worker? some communication graph?)

Early Clause Sharing Portfolios

ManySAT (Hamadi, Jabbour, and Sais 2009) [11]

- Hand-crafted diversification of four solver configurations
  — Restart policy, variable + polarity selection heuristic, ...
- Eager exchange of clauses of length ≤ 8 via lockless queues

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Plingeling (Biere 2010) [3]

- Portfolio over Lingeling configurations (shared-memory parallelism)
- Lazy exchange of information over "boss thread"
  — 2010: Unit clauses only
  — 2011: Unit clauses + equivalences
  — Since 2013: Unit clauses + equivalences + clauses of length ≤ 40, LBD ≤ 8
- Best parallel solver for many years

Massively parallel hardware?

Distributed computing, High-Performance Computing (HPC)

In distributed computing, several machines
(with no shared main memory) run together.
On each machine we run a number of processes,
each of which runs on a number of cores.
Processes commonly communicate by exchanging messages.

SuperMUC-NG: 6 336 nodes $\times$ 48 cores

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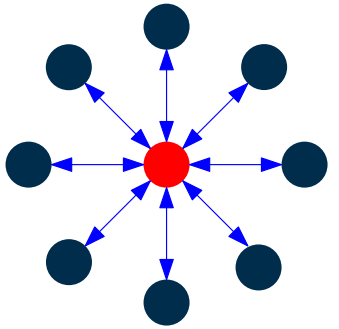
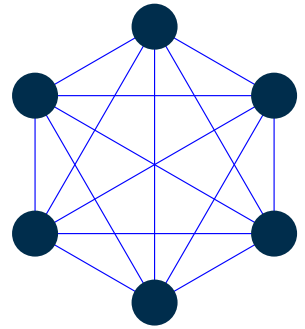
Large distributed systems (hundreds to thousands of cores) impose new requirements, challenges:

- No shared memory — communication protocols required
- Diminishing returns due to exhausted diversification of solvers
- Some exchange schemes are conceptually not scalable [7]
  - "Star graph": Master process collects, serves all exported clauses
  - Naïve (quadratic) all-to-all exchange of clauses
- HPC schedulers, administrators, committees are used to regular, easily parallelizable code with near-linear scaling





Massively parallel SAT portfolio

HordeSat (Balyo, Sanders, Sinz 2015) [1]

- Decentralization: No single leader node / process
- Two-level (“hybrid”) parallelization
  - One or several processes on each machine
  - Multiple solver threads (+ communication thread) on each process

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- Decentralization: No single leader node / process
- Two-level (“hybrid”) parallelization
  - One or several processes on each machine
  - Multiple solver threads (+ communication thread) on each process
- Diversification options:
  - Native diversification (set of hand-crafted solver configurations)
  - Modifying some initial variable phases
  - Random seeds
- Periodic all-to-all clause exchange

Clause Exchange in HordeSat

HordeSat's sharing logic

- Only clauses with LBD ≤ 2 are considered for sharing
  - Constraint is lifted successively if processes under-produce
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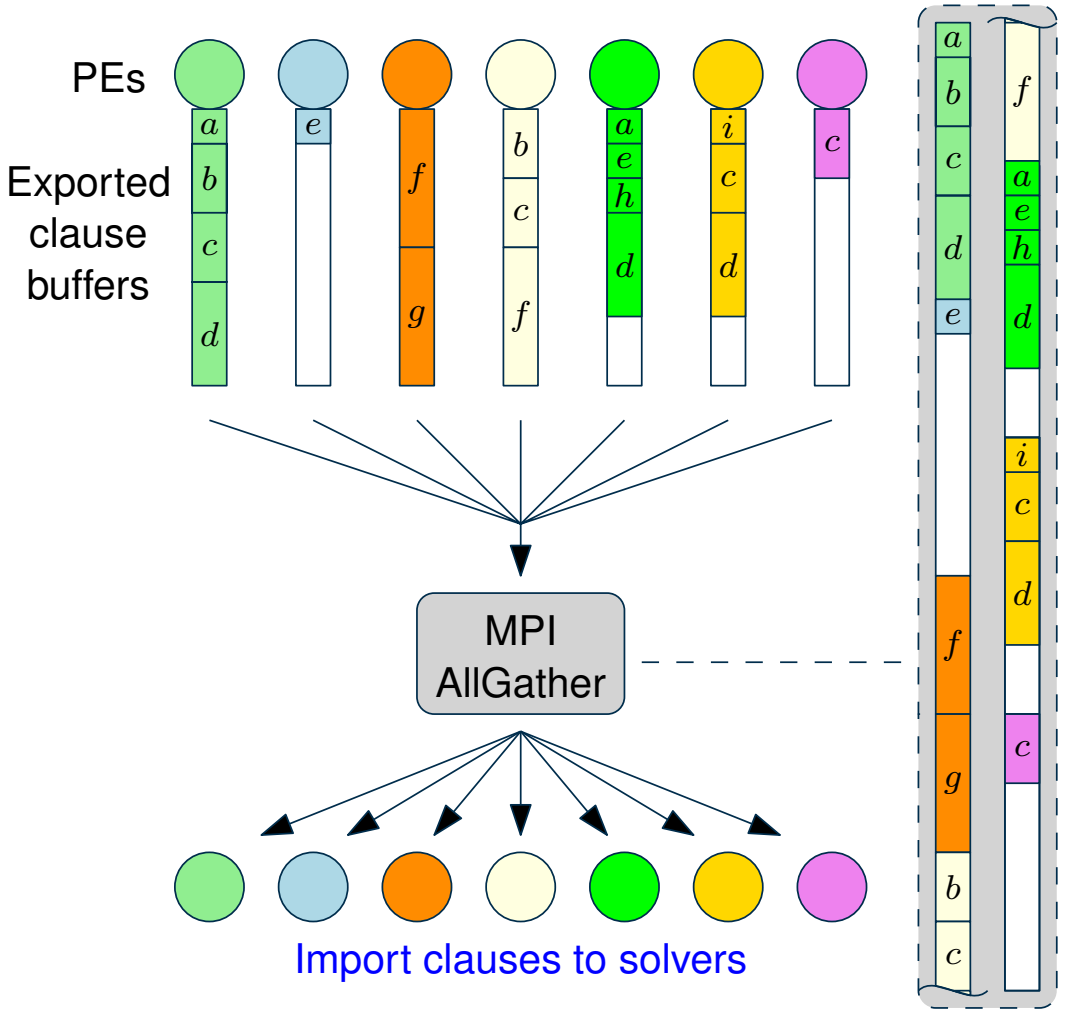
- Each process' best clauses are shared with everyone
  - Limited to 1500 clause literals per process

$\Rightarrow$ Concatenation of p produced clause buffers

- Approximate, post-hoc filtering of clauses



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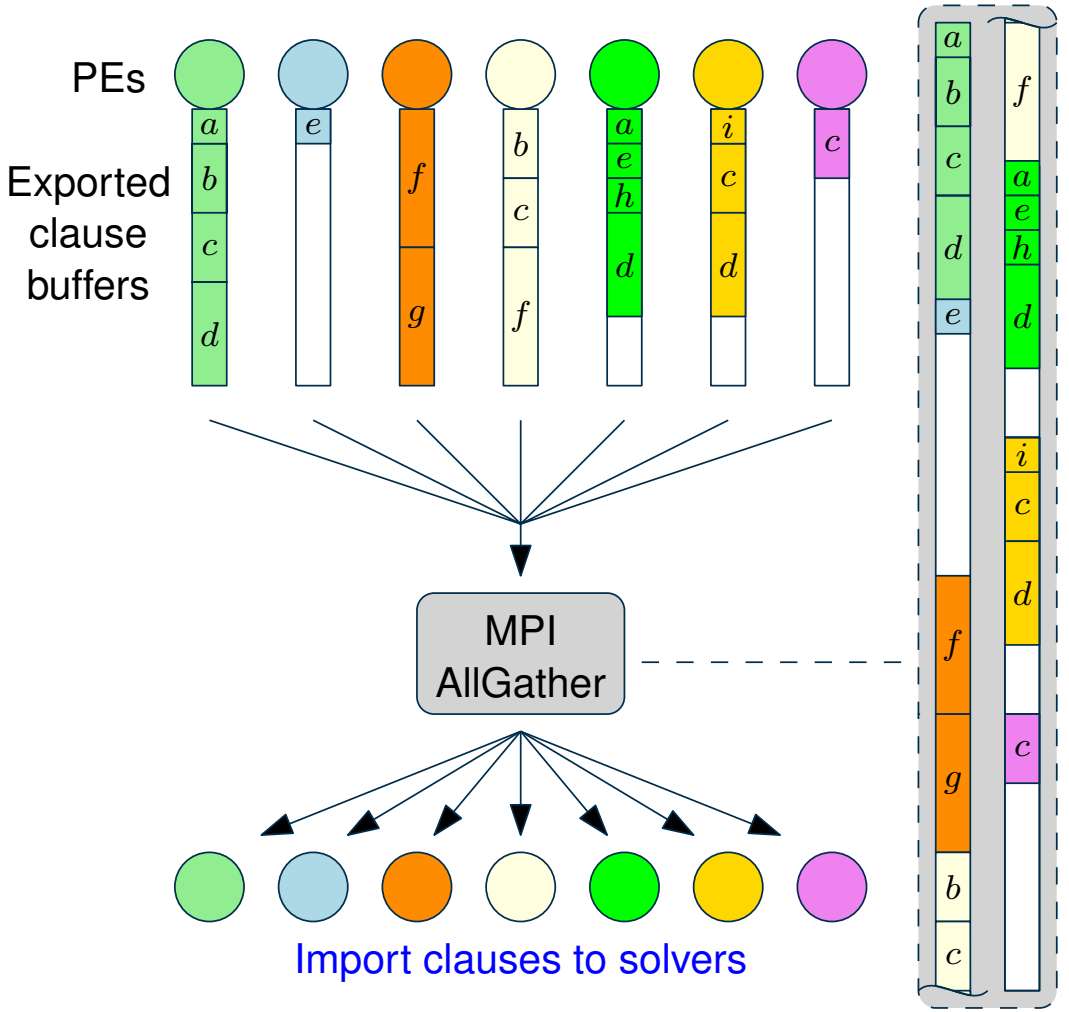

Issues:

- Many ("high" LBD) clauses are not shared but discarded
- "Holes" in buffer carrying no information
- Duplicate clauses
- Buffer grows proportionally with # proc.
  $\Rightarrow$ Bottleneck w.r.t. communication, local work

HordeSat: Results

- Super-linear speedups for individual instances
  = speedup $> c$ on c cores! How?

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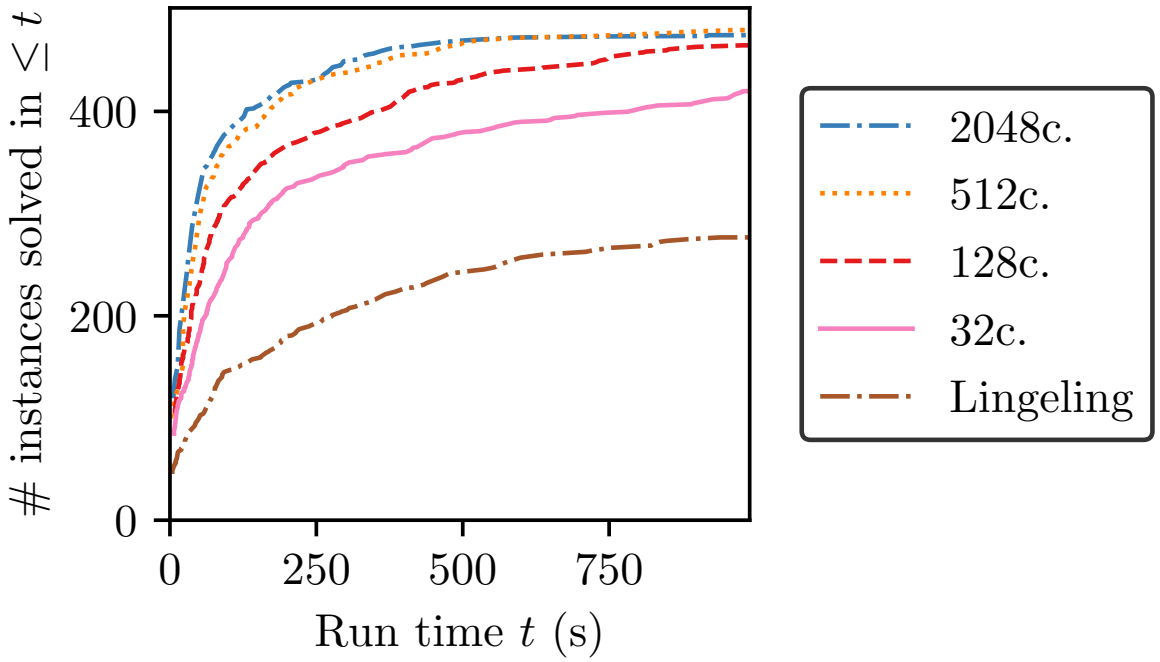
  - SAT: "NP luck" – some solver got lucky
  - UNSAT: distributed memory accommodates more clauses than any sequential solver
  - General: sequential schedule of parallel algorithm may outperform sequential algorithm!
- Median speedup: 3 at 16 cores, 11.5 at 512 cores
  - Efficiency: $11.5/512 \approx 2.2\%$
  - Deploying HordeSat is often not worth it
- No improvement beyond $\approx$ 500 cores



Data extracted from HordeSat paper [1]

From HordeSat to MallobSat

Research Question

How can we improve performance, (resource-)efficiency, and average response times of SAT solving in modern distributed environments?

From HordeSat to MallobSat

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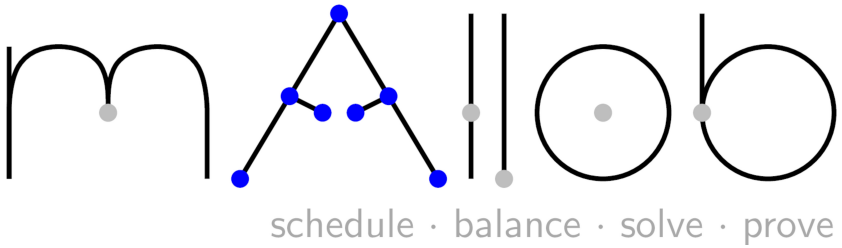
How can we improve performance, (resource-)efficiency, and average response times of SAT solving in modern distributed environments?

Result: Mallob

Mallob is a platform for SAT solving (*and other NP-hard problems*) with:

- multi-user, on-demand, malleable scheduling and solving of many problems at once [19]
- distributed SAT engine **MallobSat**: the HordeSat paradigm re-engineered and made efficient [24]
- state-of-the-art SAT performance from dozens to thousands of cores [23]



https://satres.kikit.kit.edu/research/mallob

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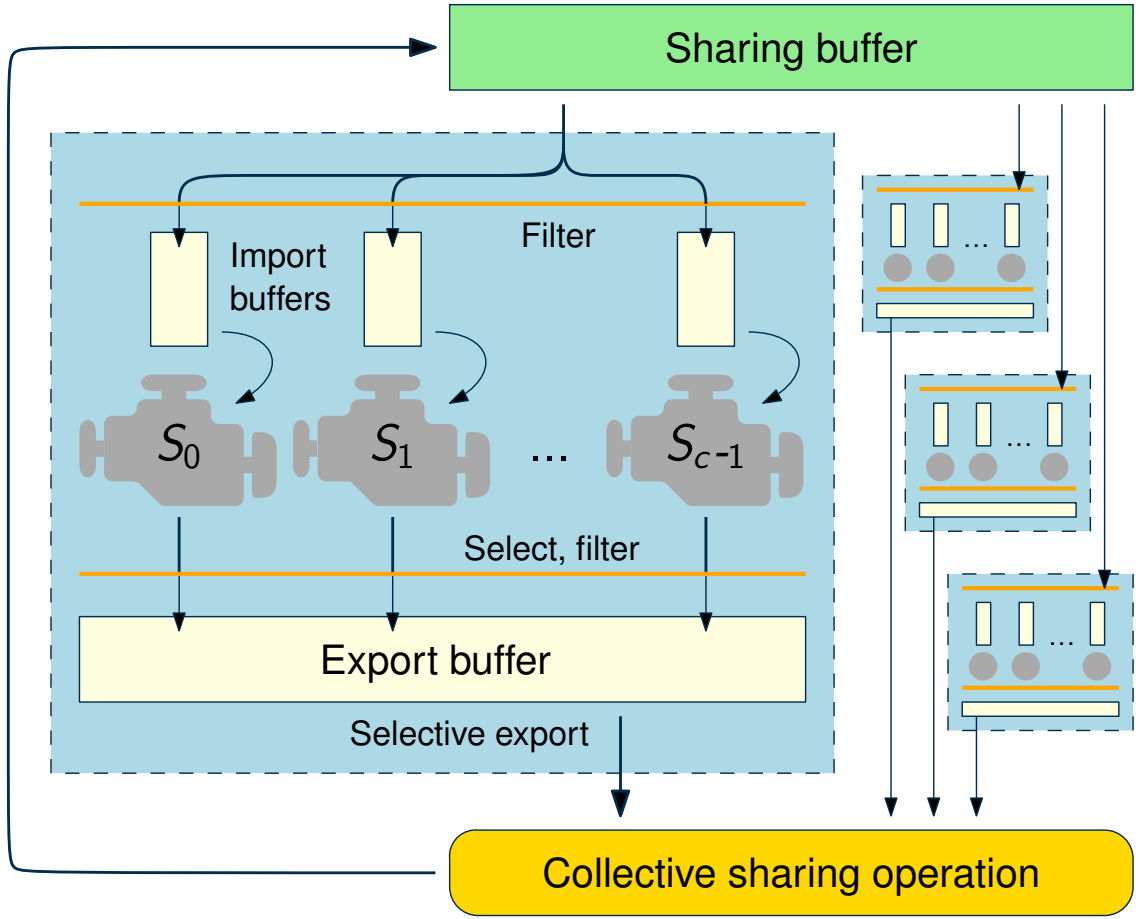

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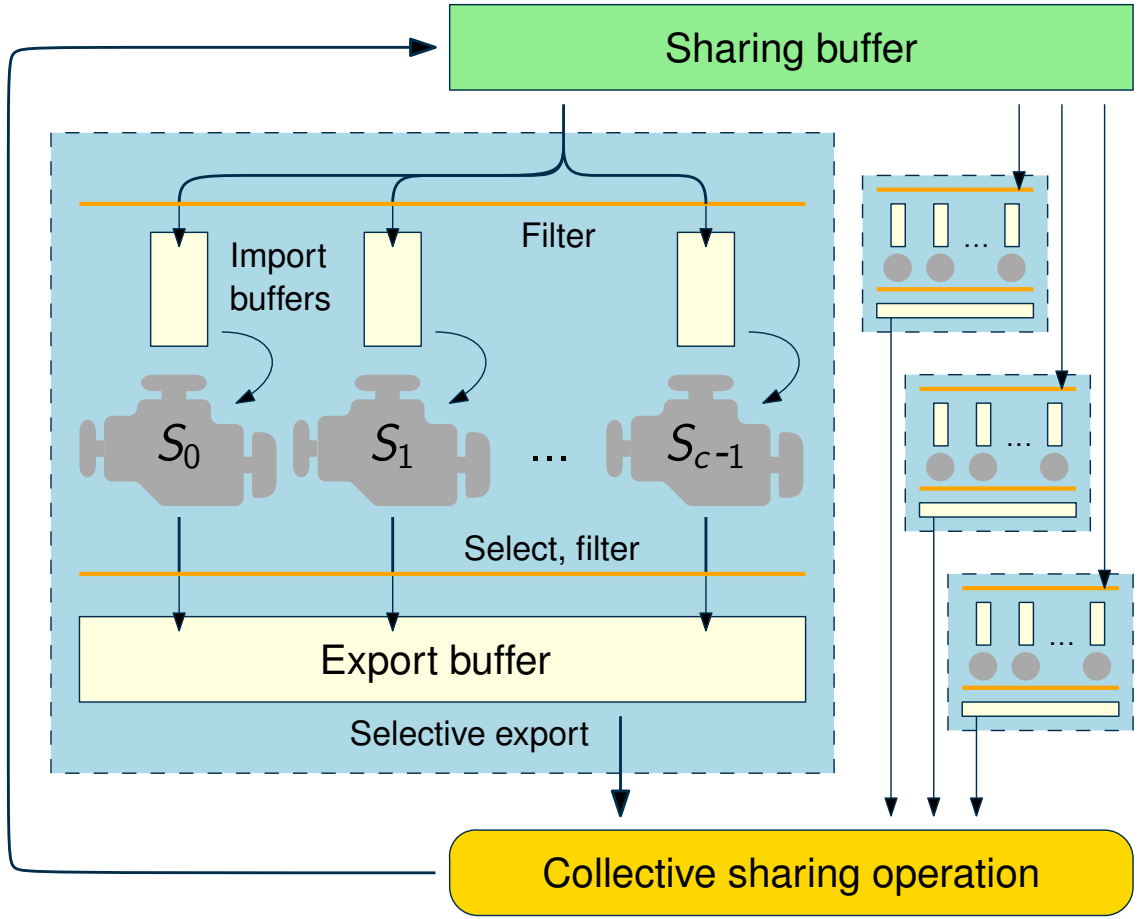

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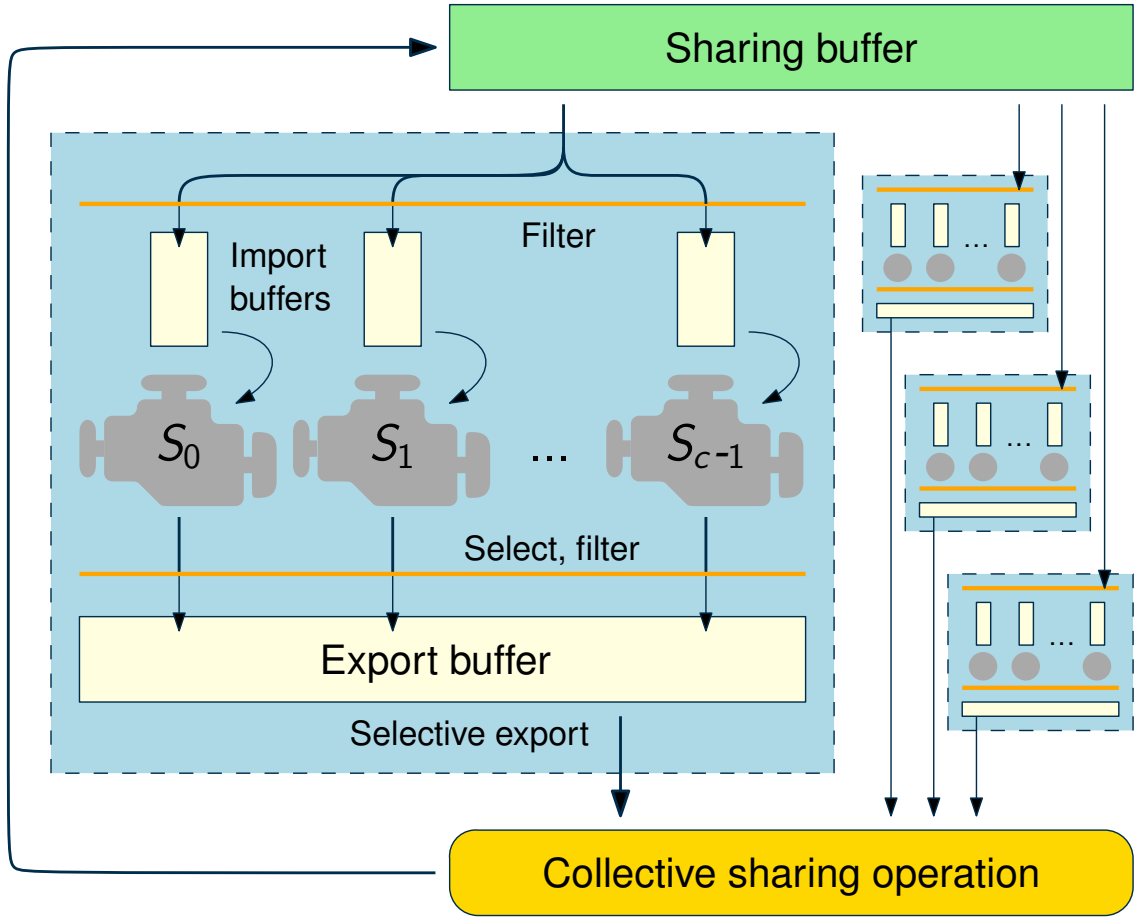

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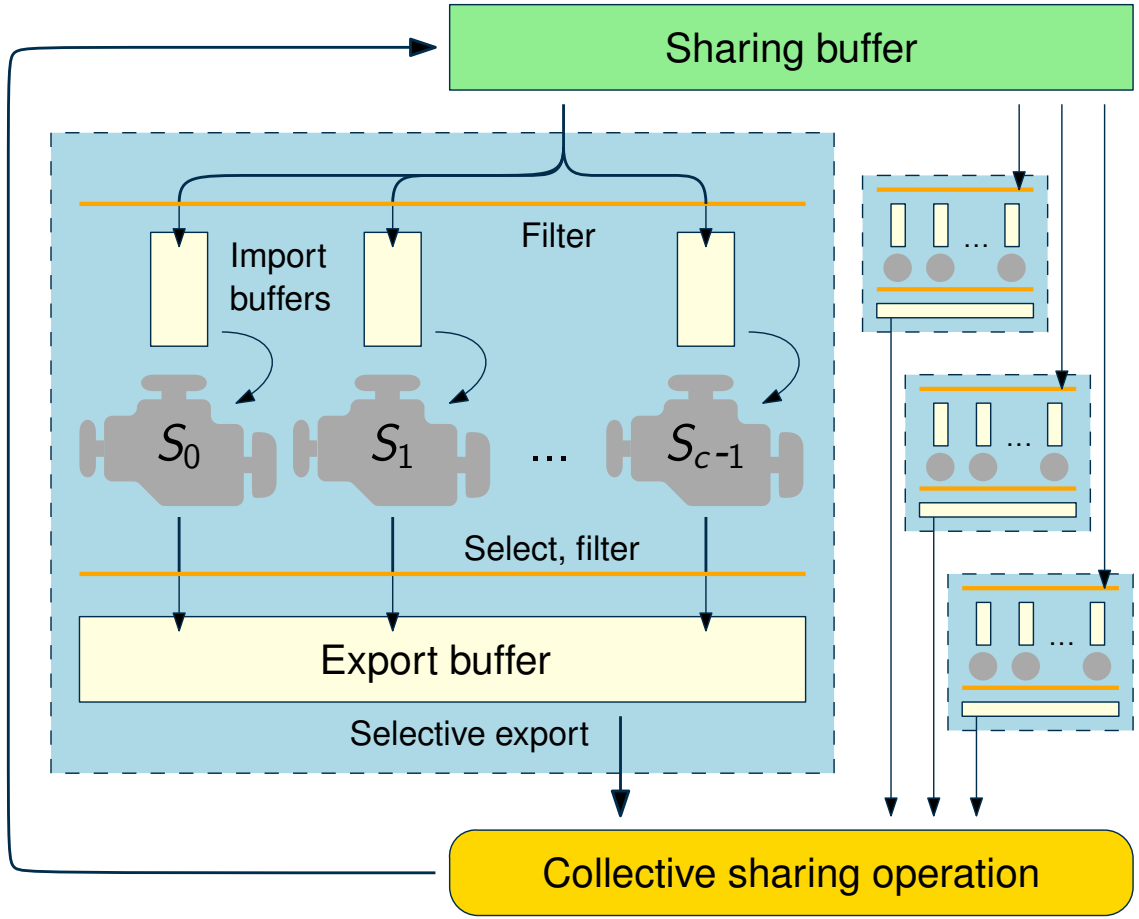

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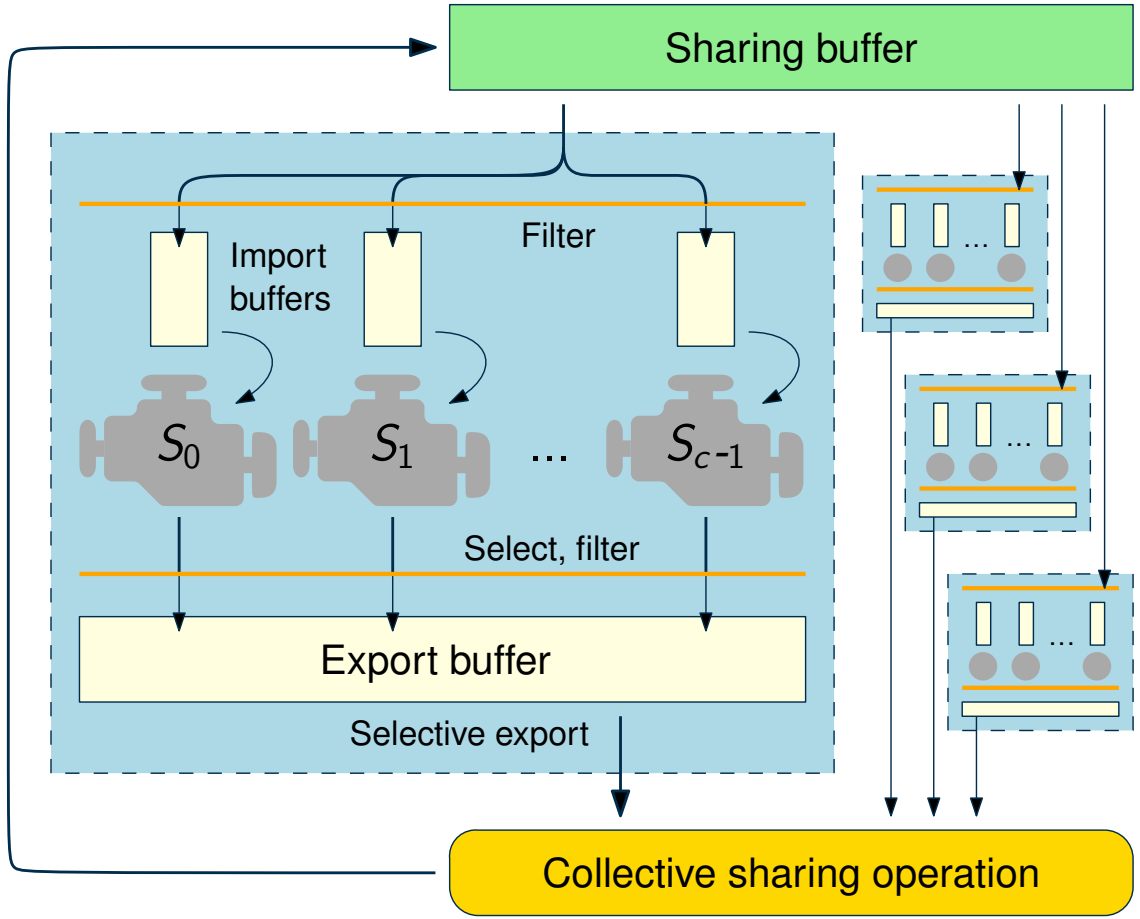

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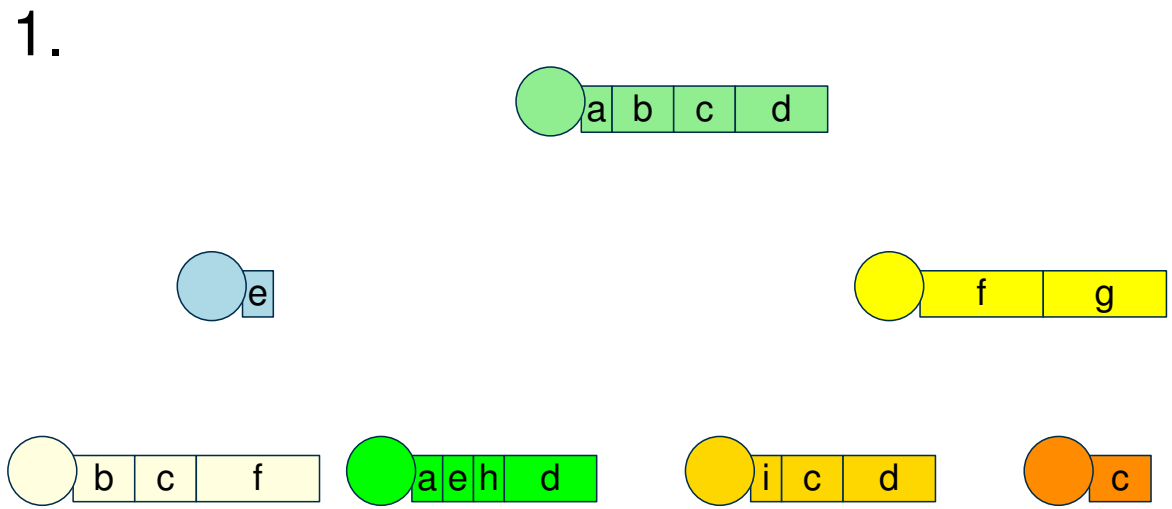

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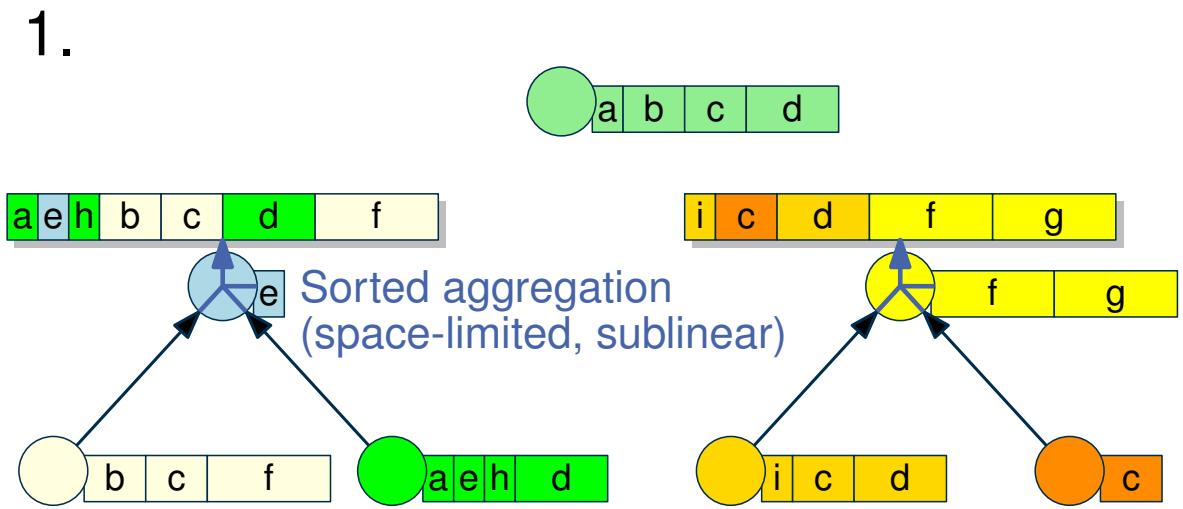

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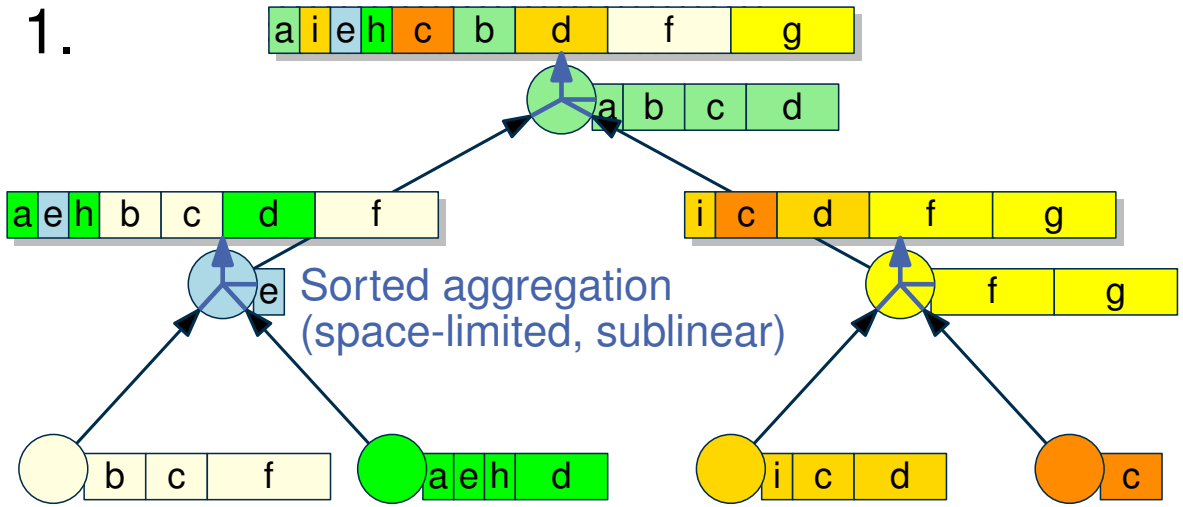

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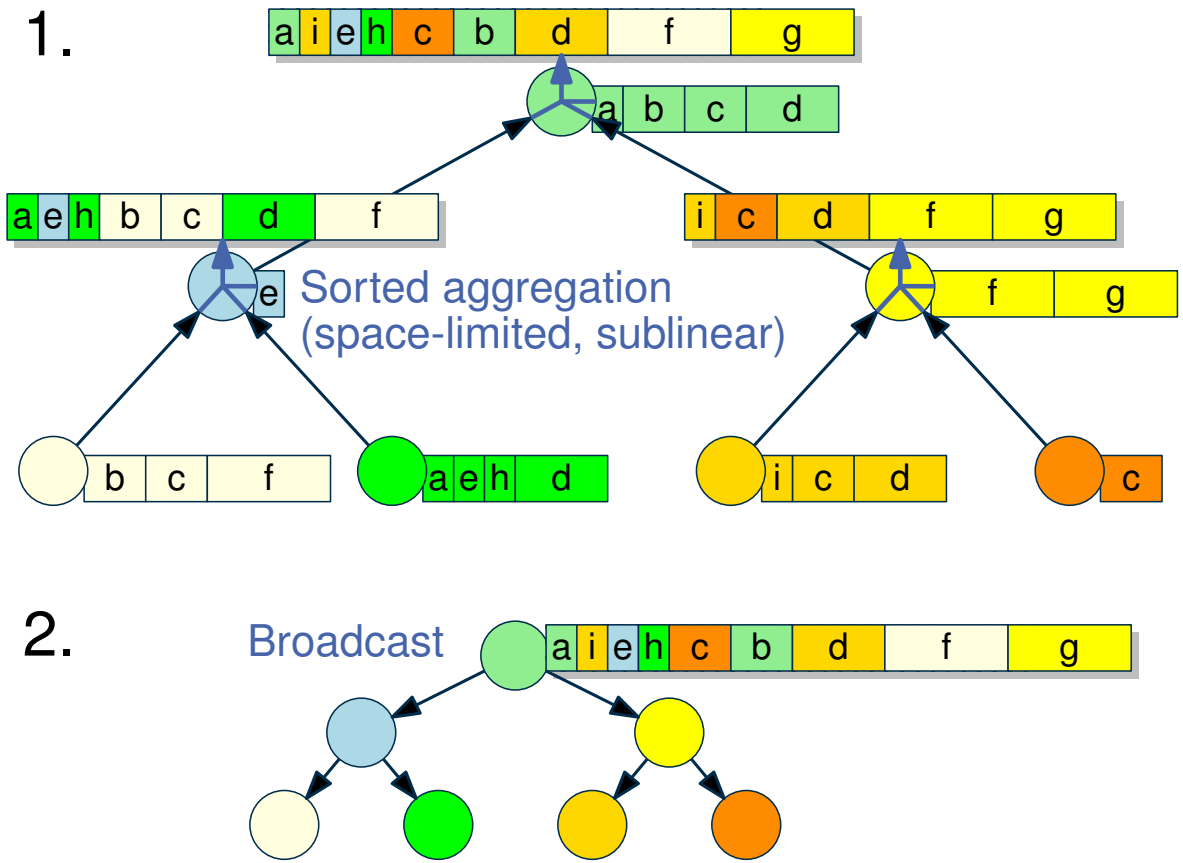

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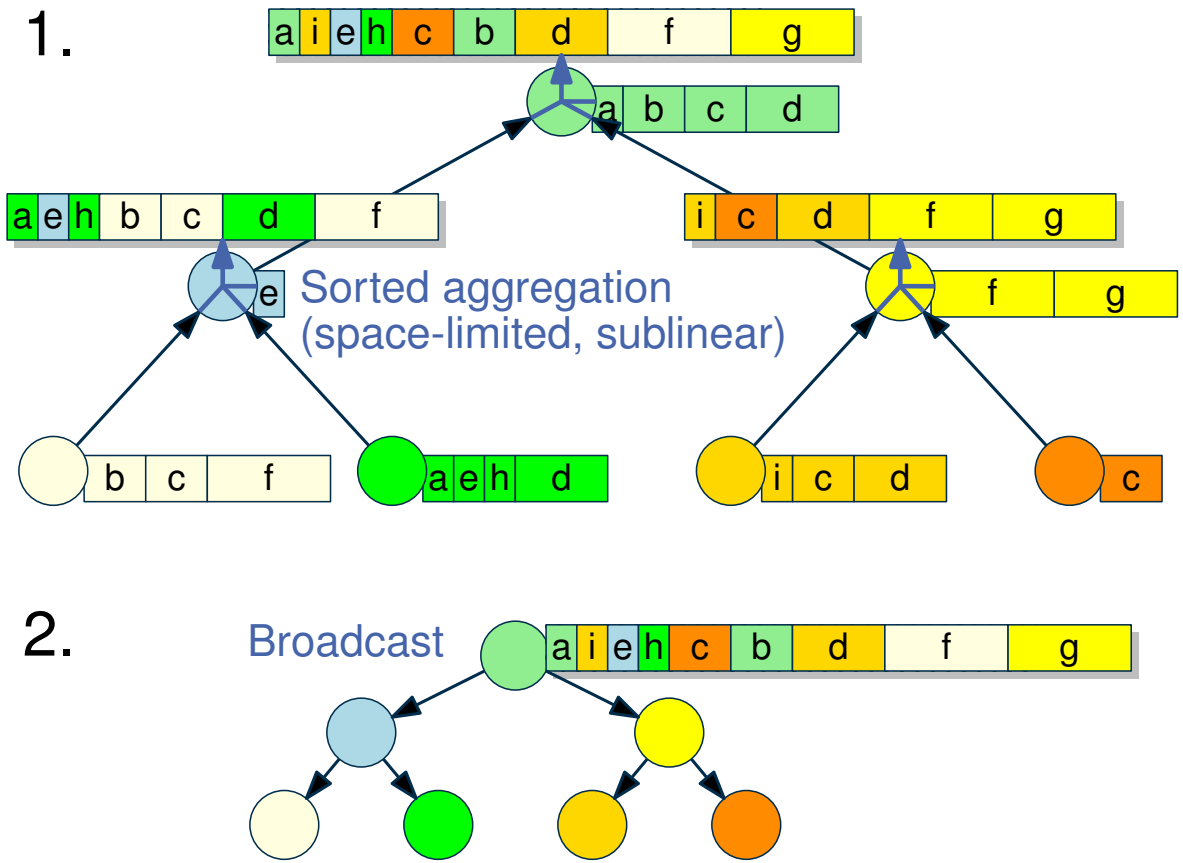

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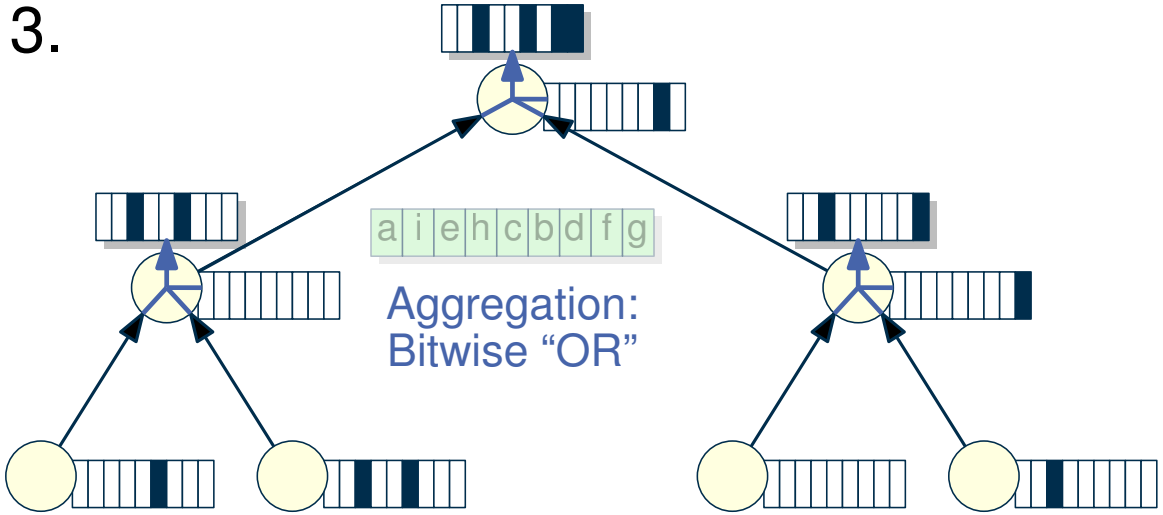

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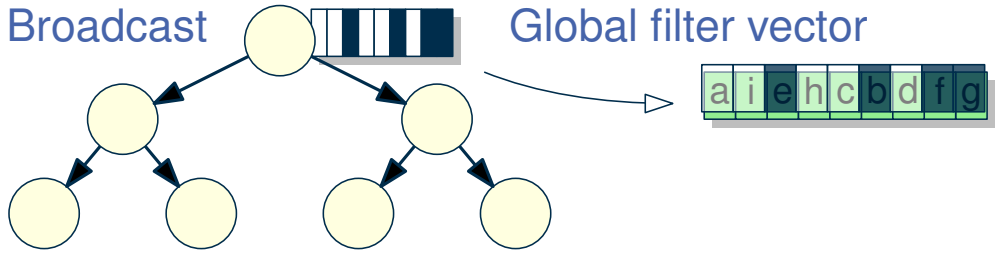

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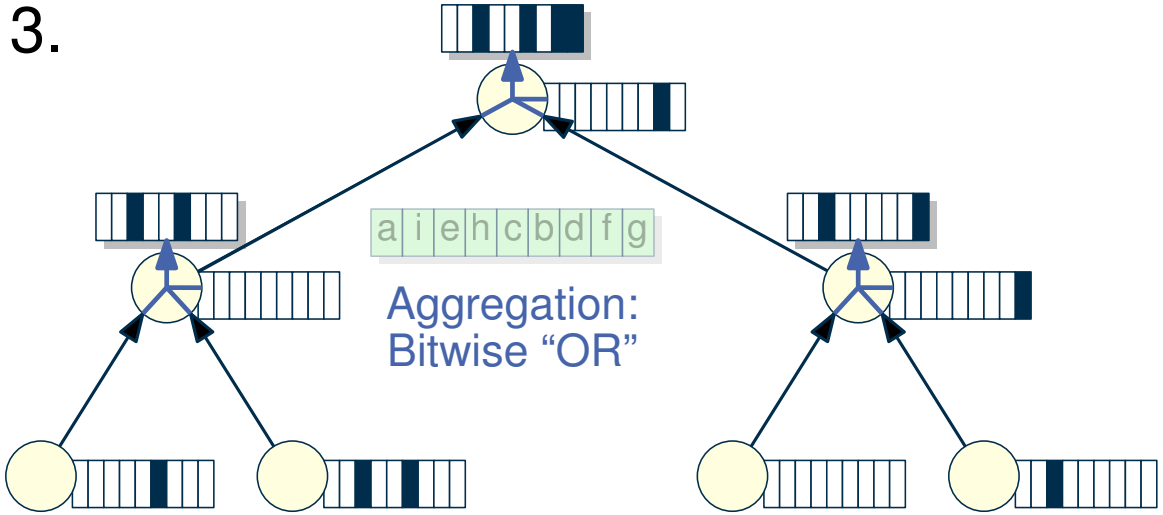

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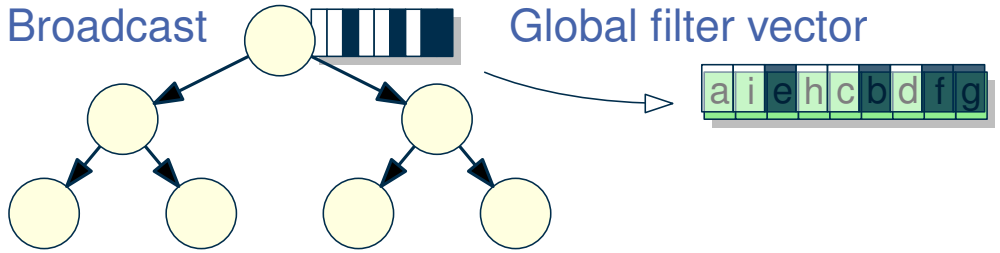

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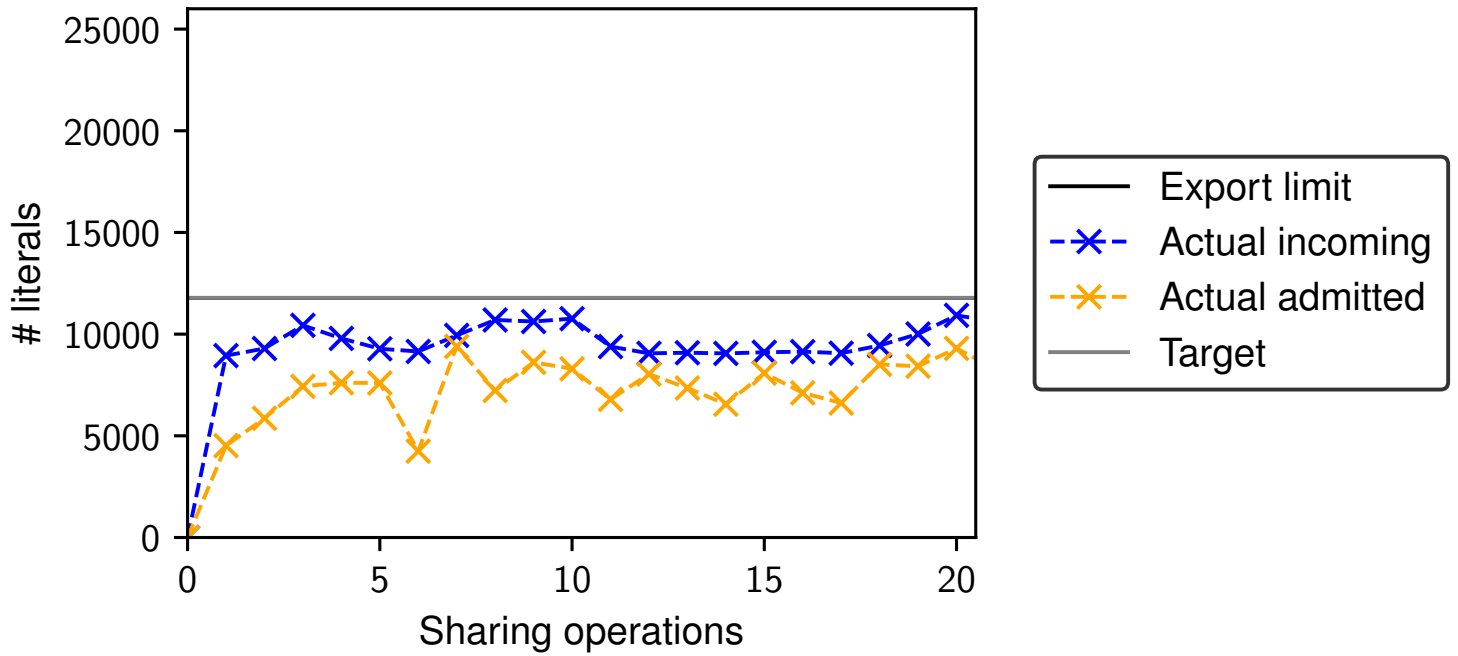

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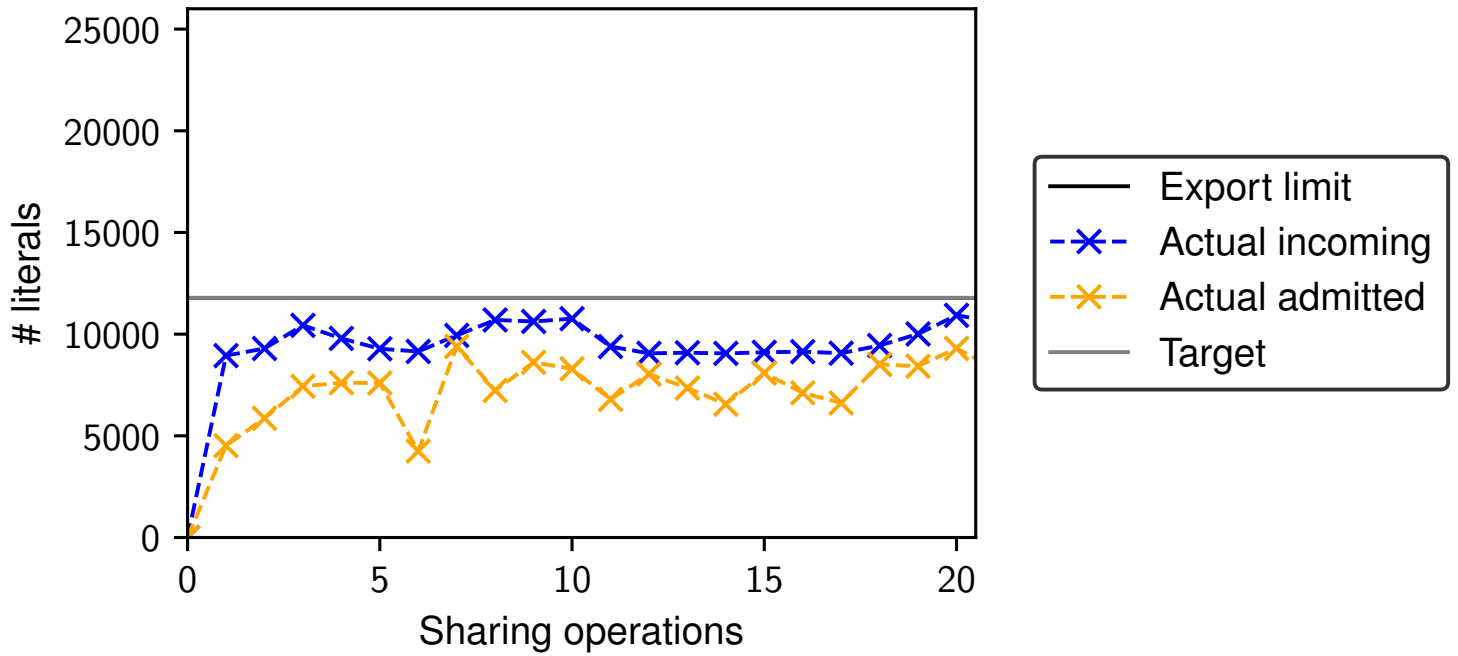

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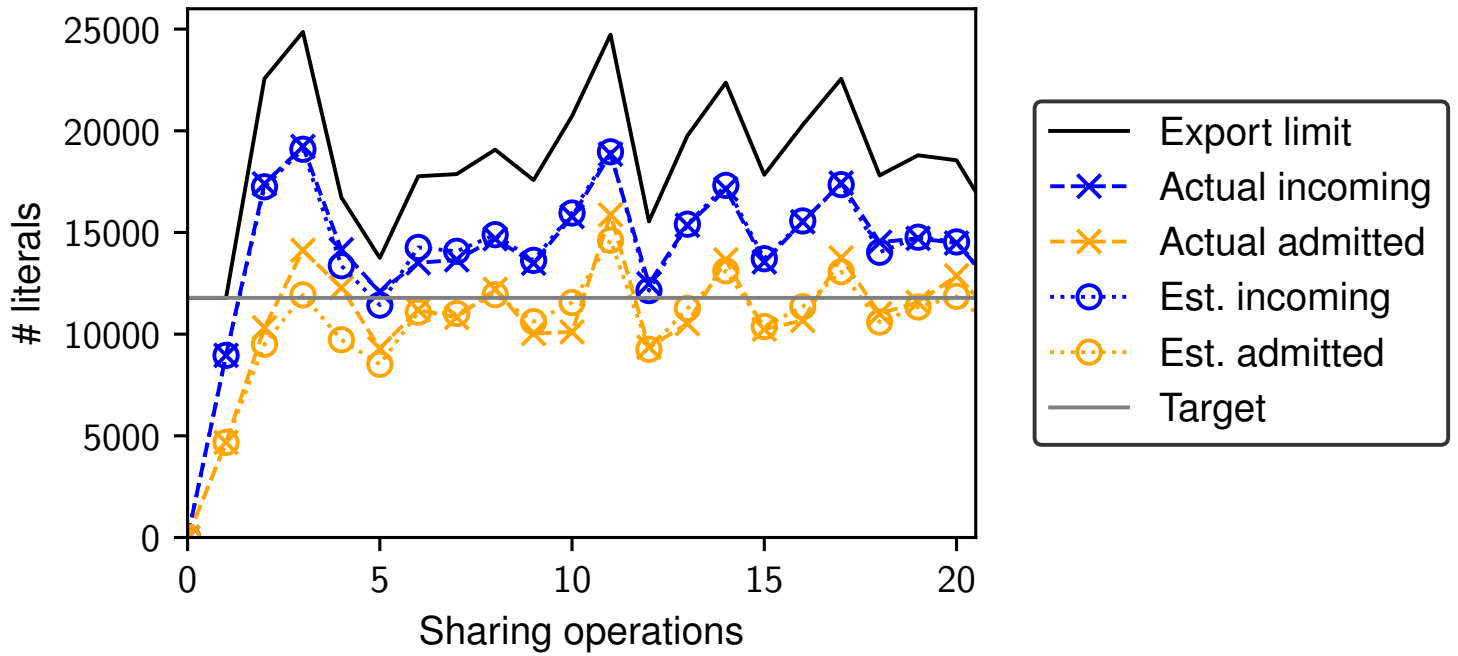

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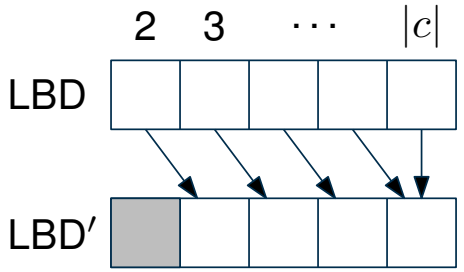

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768 cores $\times$ 349 instances $\times$ 300 s

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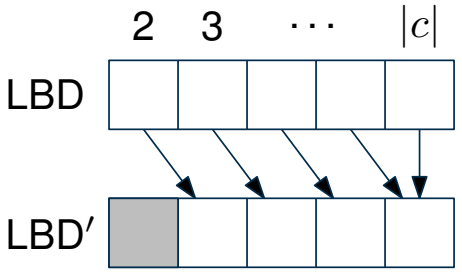

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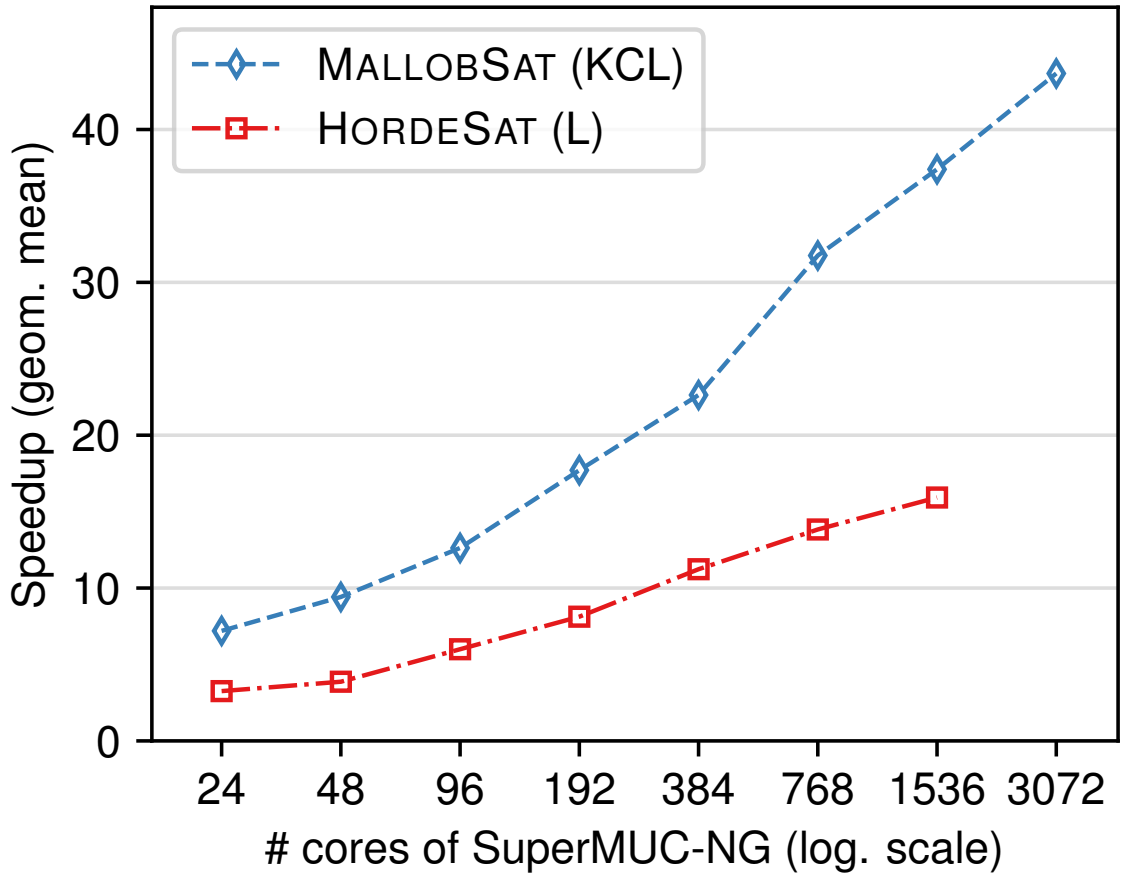
Scaling of MallobSat [23]



400 problems from SAT Comp. 2021 · Seq. baseline KissAT_MAB-HyWalk · Seq. limit 32 h (331 solved) · Par. limit 300 s

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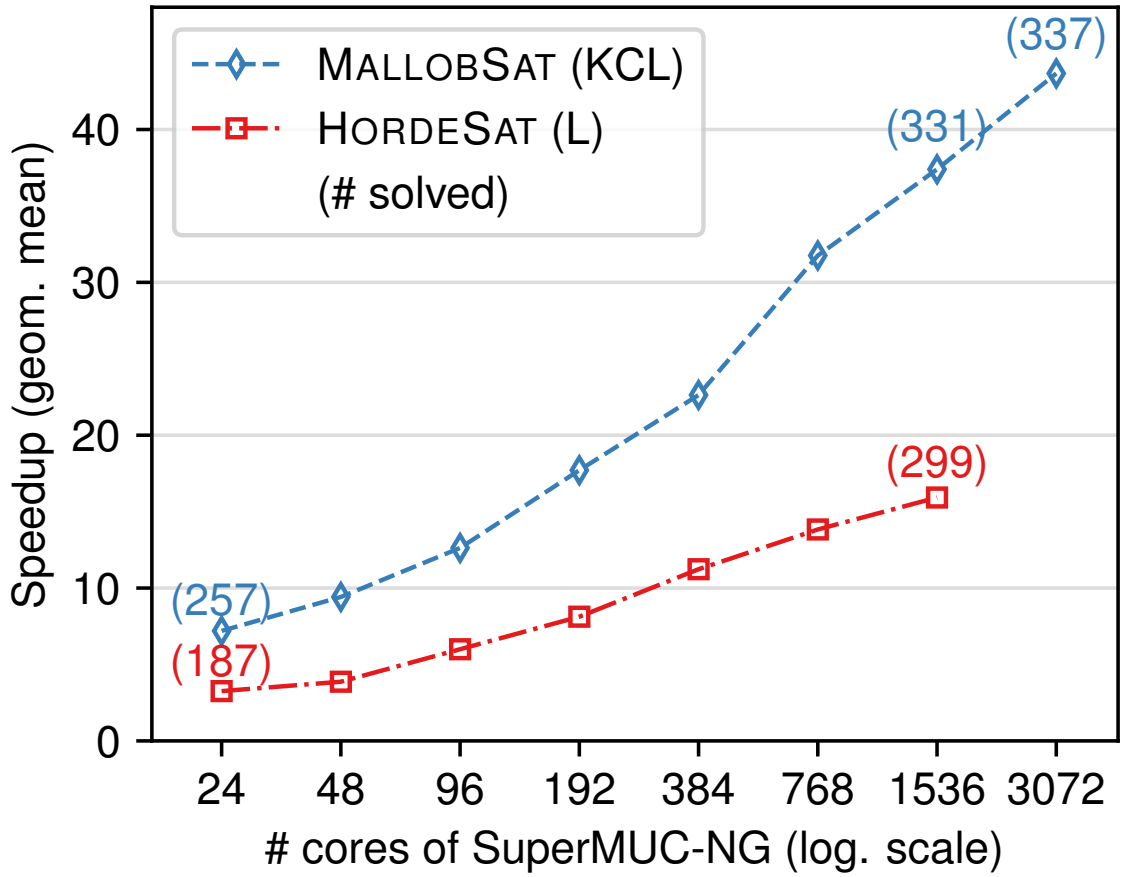

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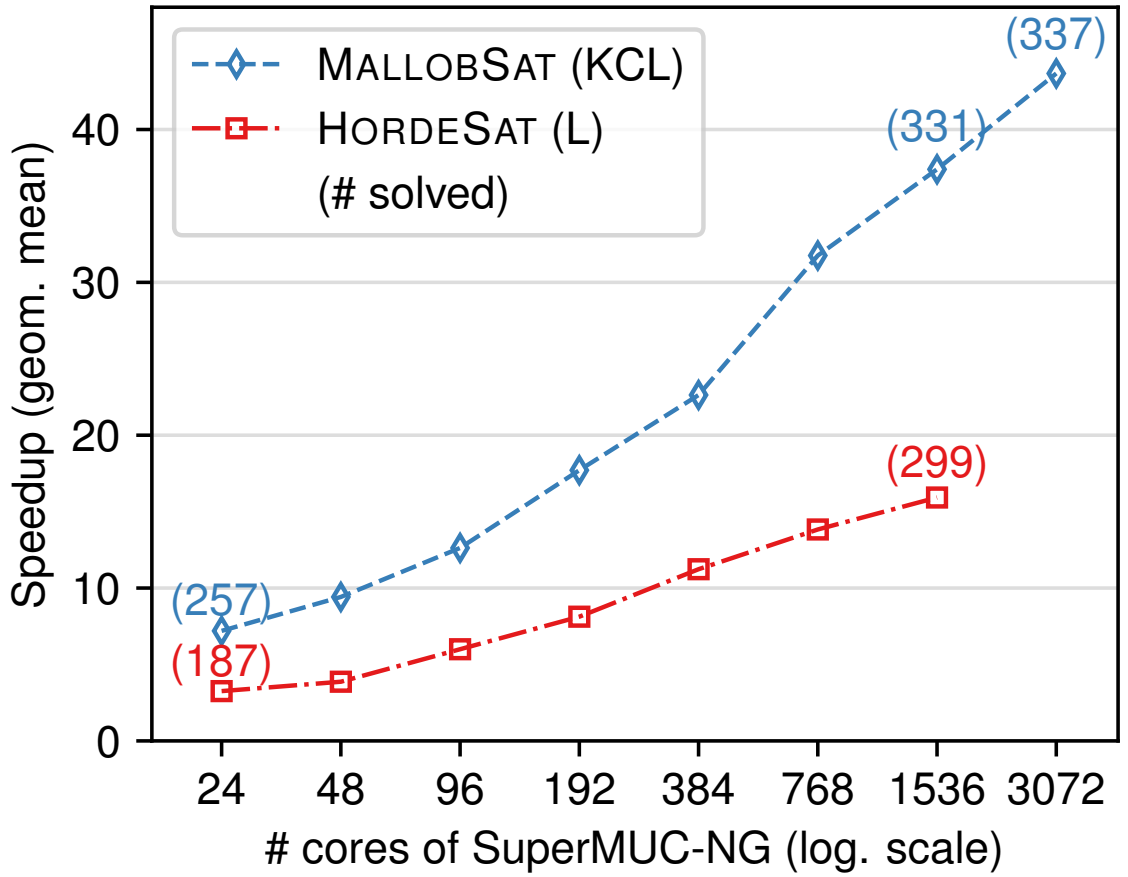

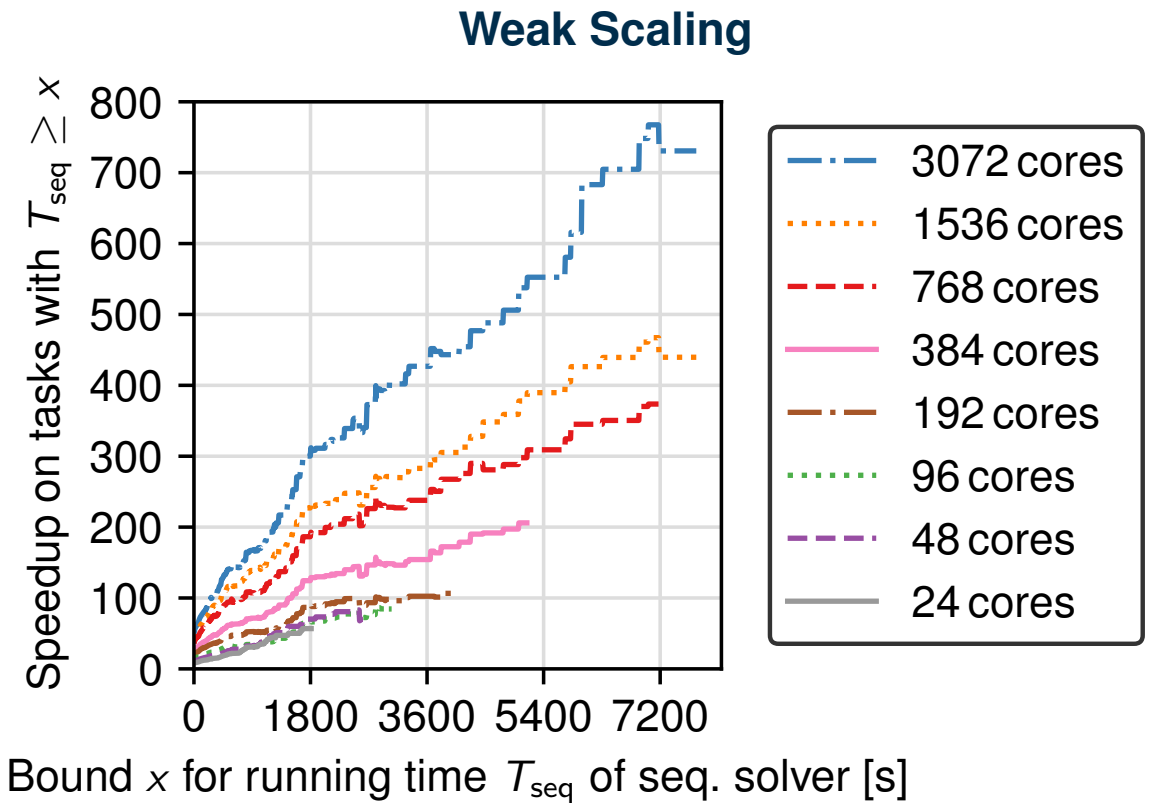

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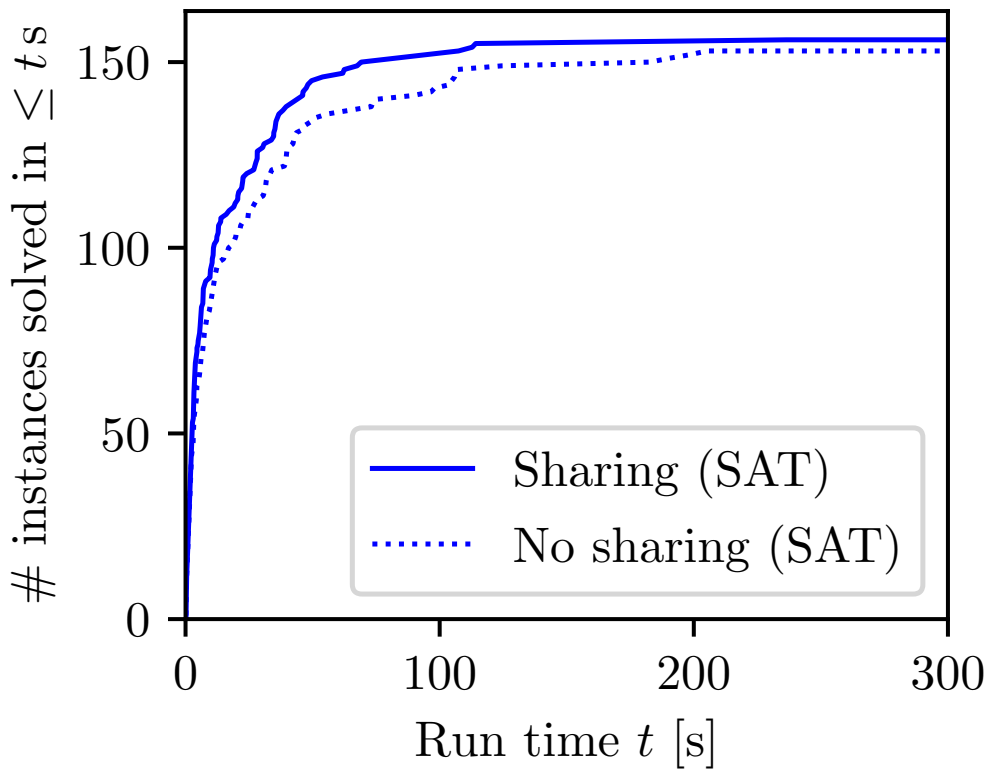
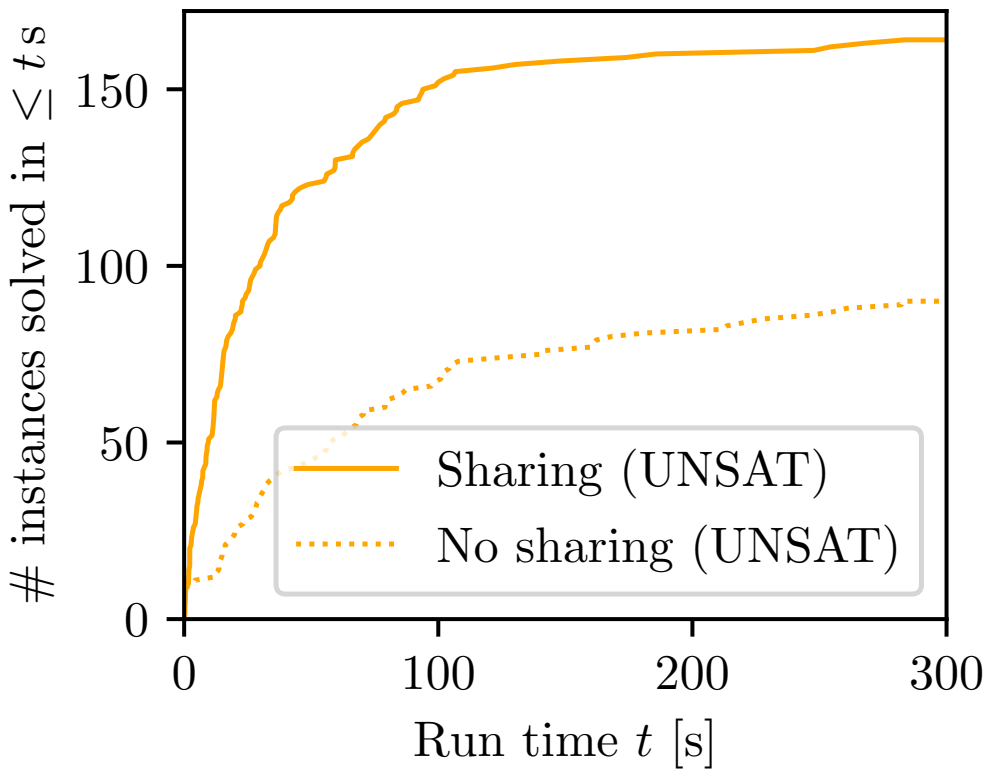
Merit of Clause Sharing, SAT vs. UNSAT





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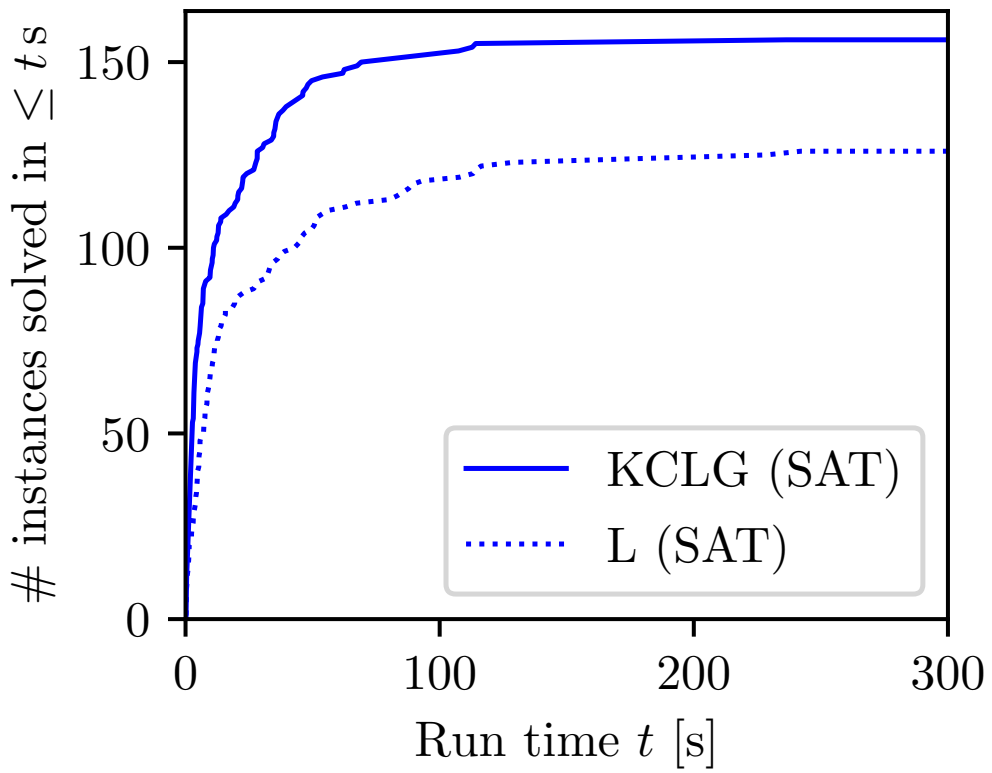
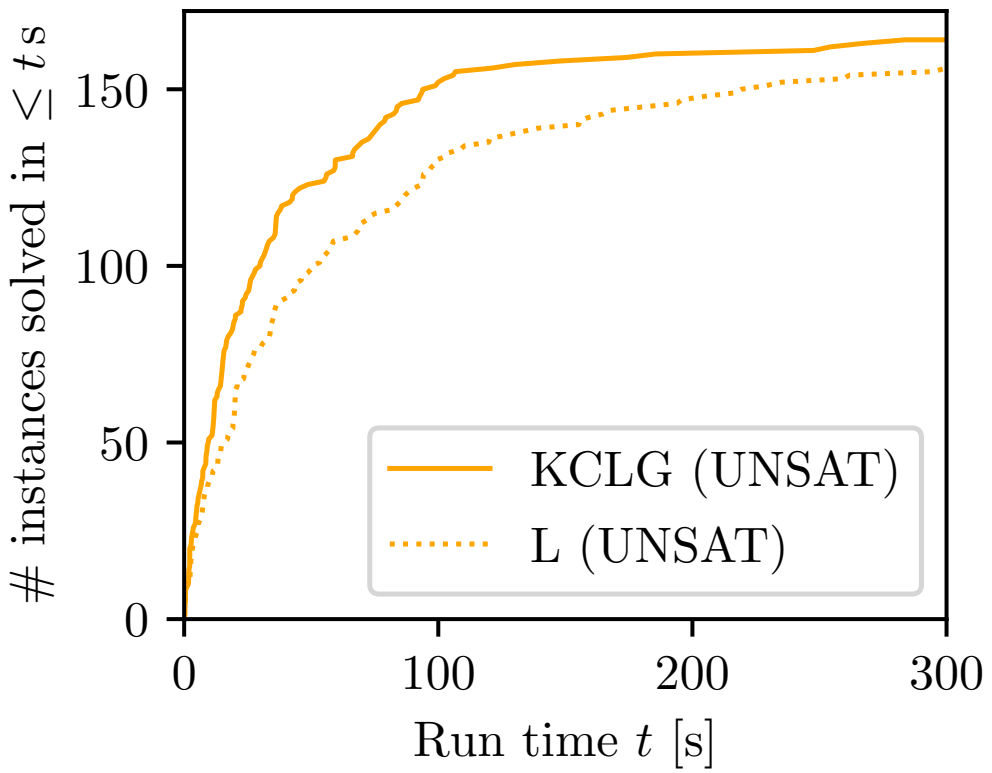
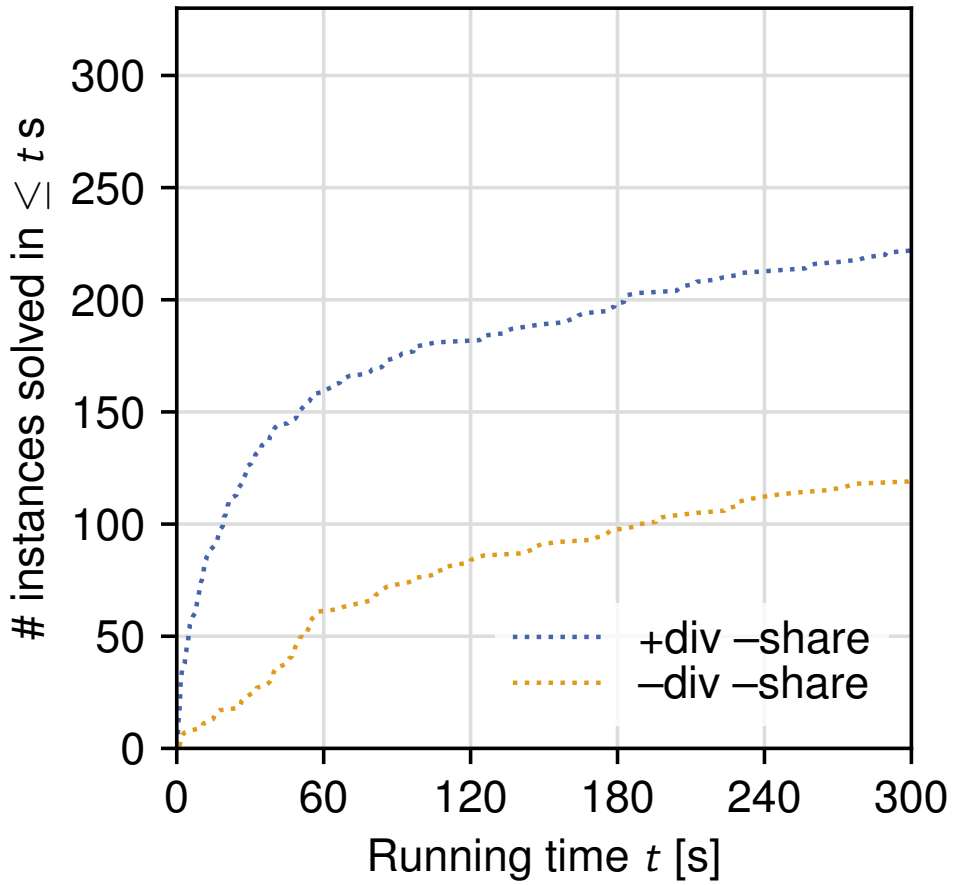




768 cores $\times$ 349 “solvable” instances from ISC 2022 $\times$ 300 s, with clause sharing

Diversification vs. Sharing @ 768 Cores [23]



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349 problems from SAT Comp. 2022 · CaDiCaL only

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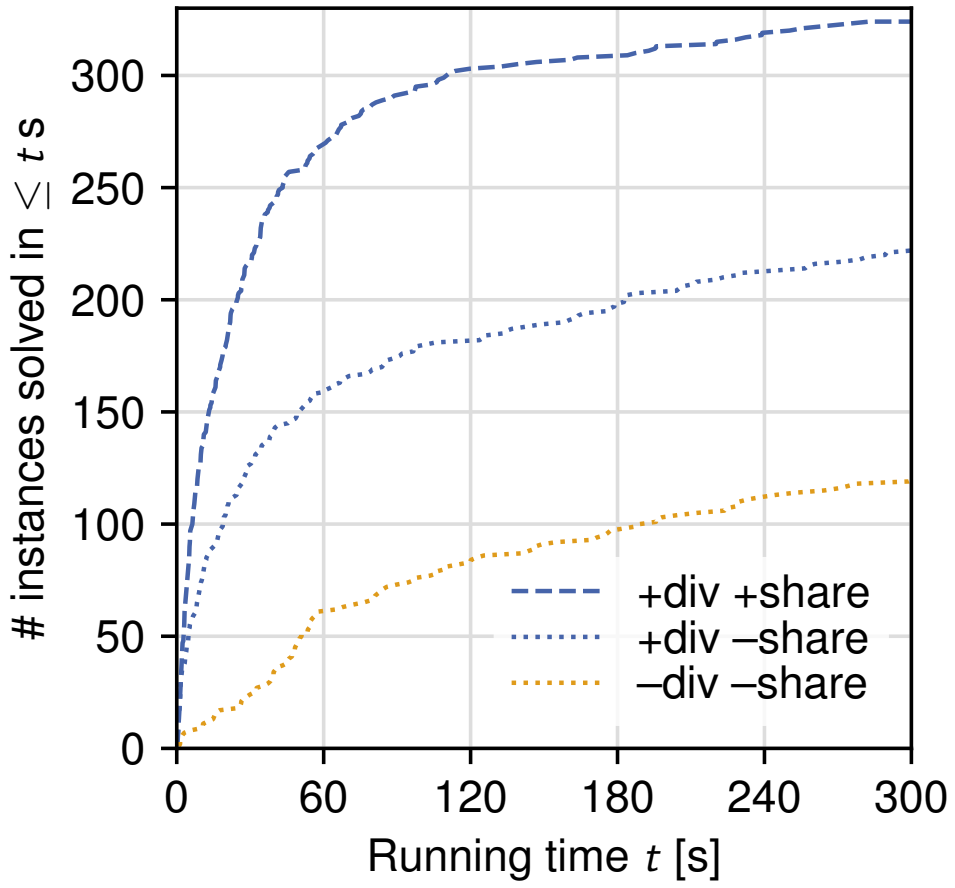

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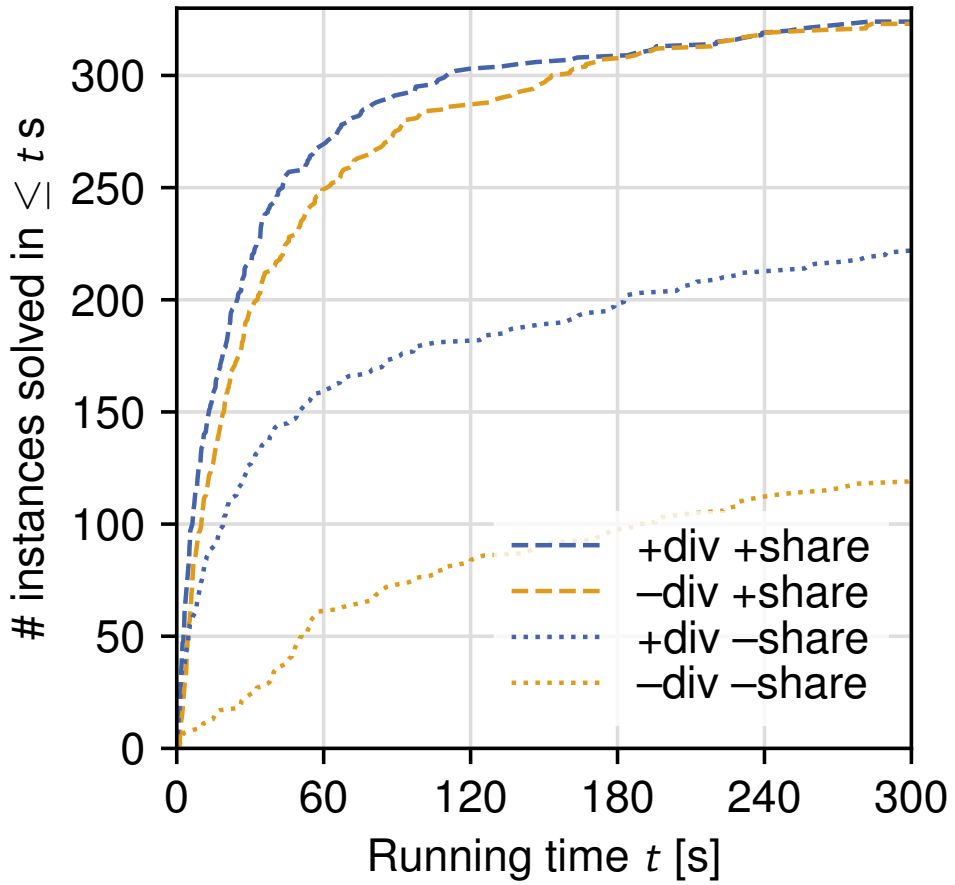

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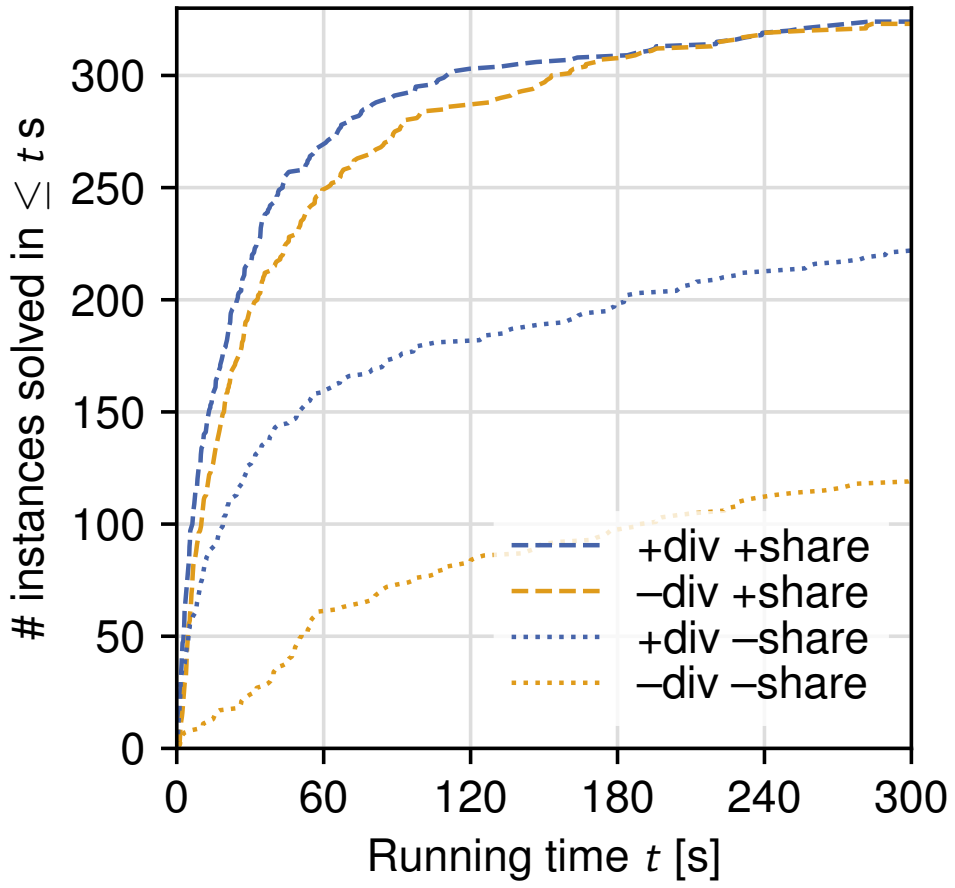

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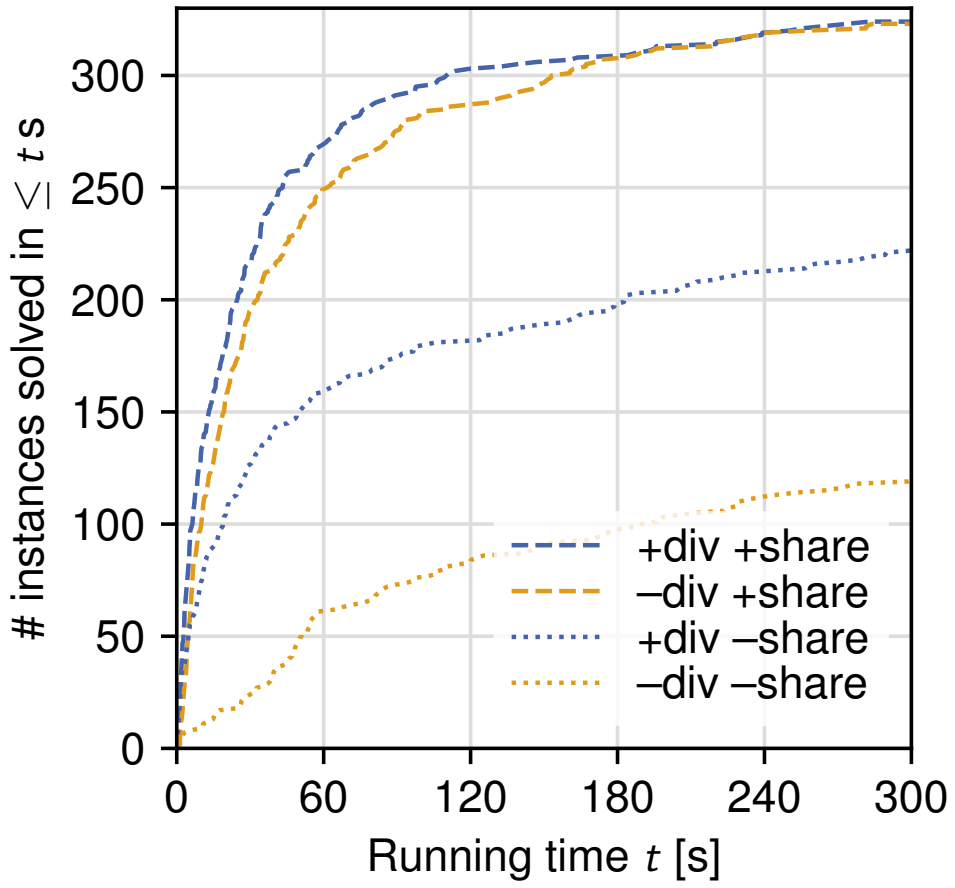

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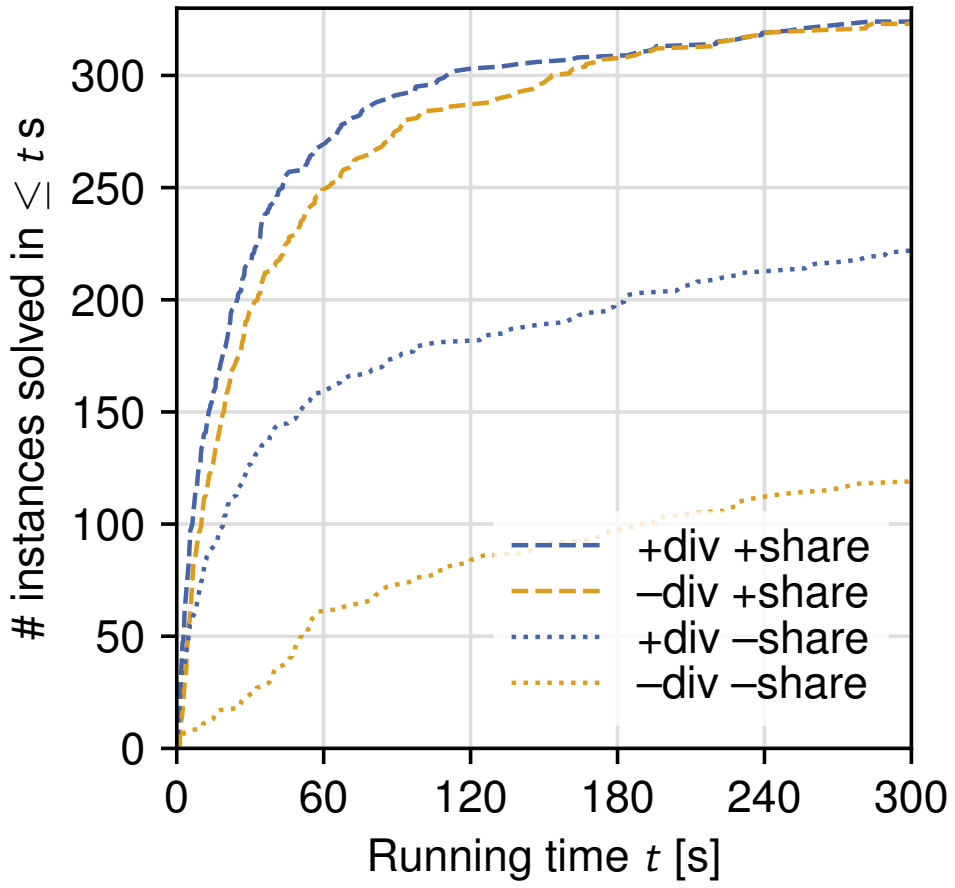

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⇒ Portfolio of diverse configurations is dispensable

⇒ Clause sharing is essential

MallobSat: A Portfolio Solver?

Prevalent concept in literature: **Portfolio solver** *with clause sharing* / Clause-sharing portfolio

- "*a problem instance is independently given to a collection of solvers competing for a solution in parallel*"
  – Fichte et al., 2023 [9]
- "*each thread runs a different SAT solver on the same instance[, which] in combination with clause-sharing leads to surprisingly good performance for small portfolio sizes*"
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Our view, based on empirical observations:

MallobSat is a Clause-sharing solver *with diversification*

- Clause sharing = main driver of scalability
- Adding explicit diversification is beneficial but not essential
- Applicability to other solvers?

Better Efficiency?

Massive parallelism for a single formula

- Faster solving times
- Can resolve problems out of reach for sequential solvers
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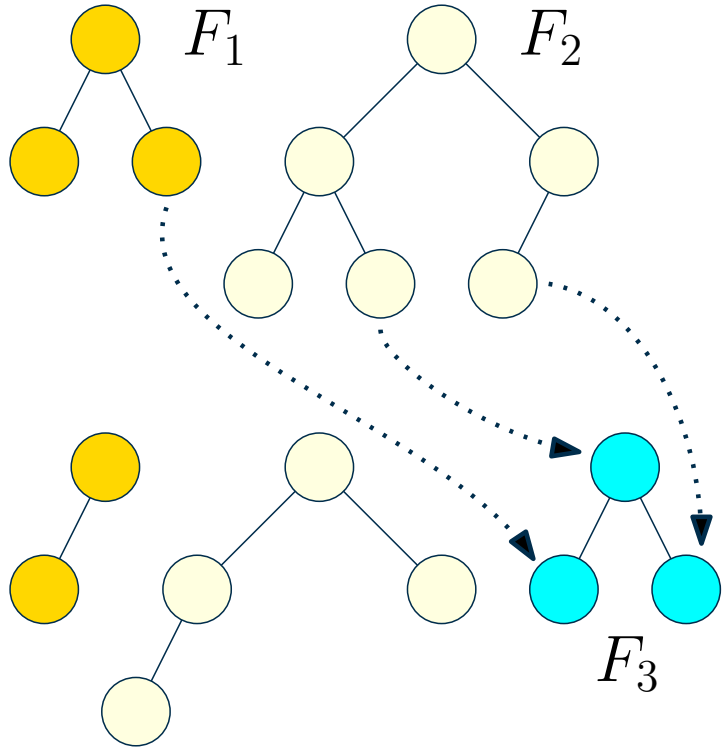
Best of both worlds? [19]

- On demand scheduling of incoming (SAT) jobs
- Resize jobs during their execution as needed
- Few milliseconds to schedule an incoming job,
  full utilization whenever sufficient demand is present



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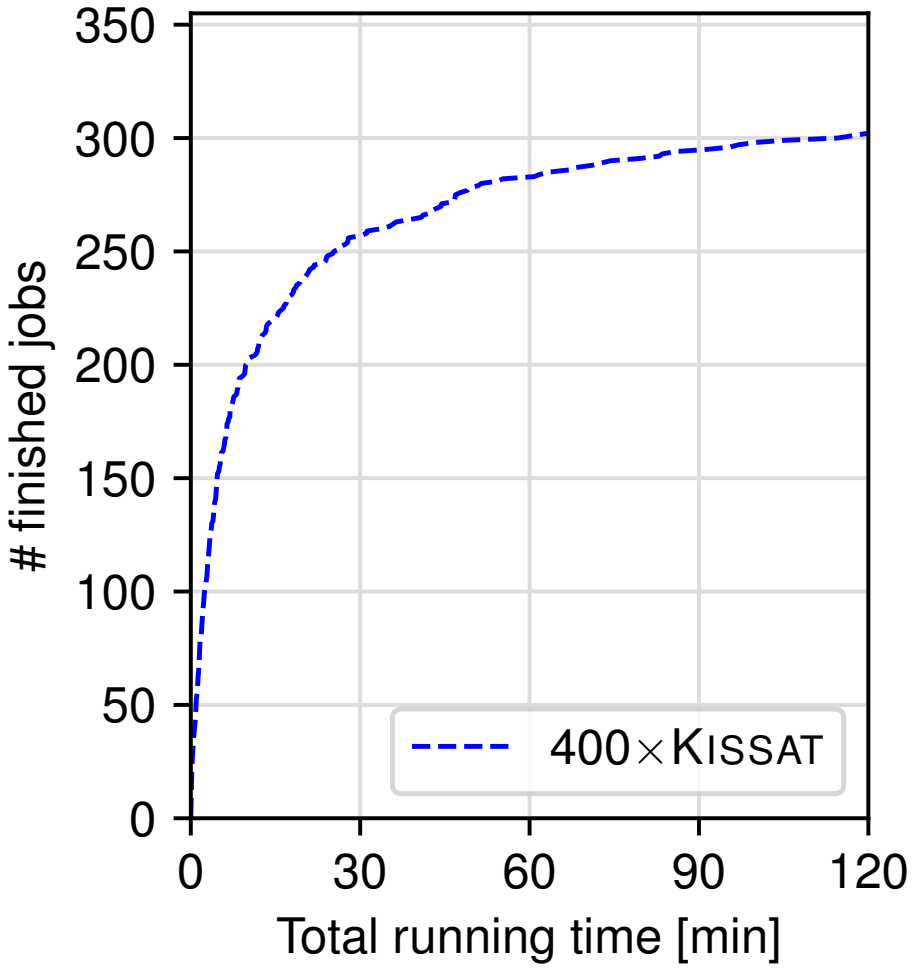
Extreme 1: 400 sequential solvers

- "Embarrassingly parallel" job processing
- Highly efficient (no redundant work)
- Sequential solving only $\rightarrow$ no scaling opportunity



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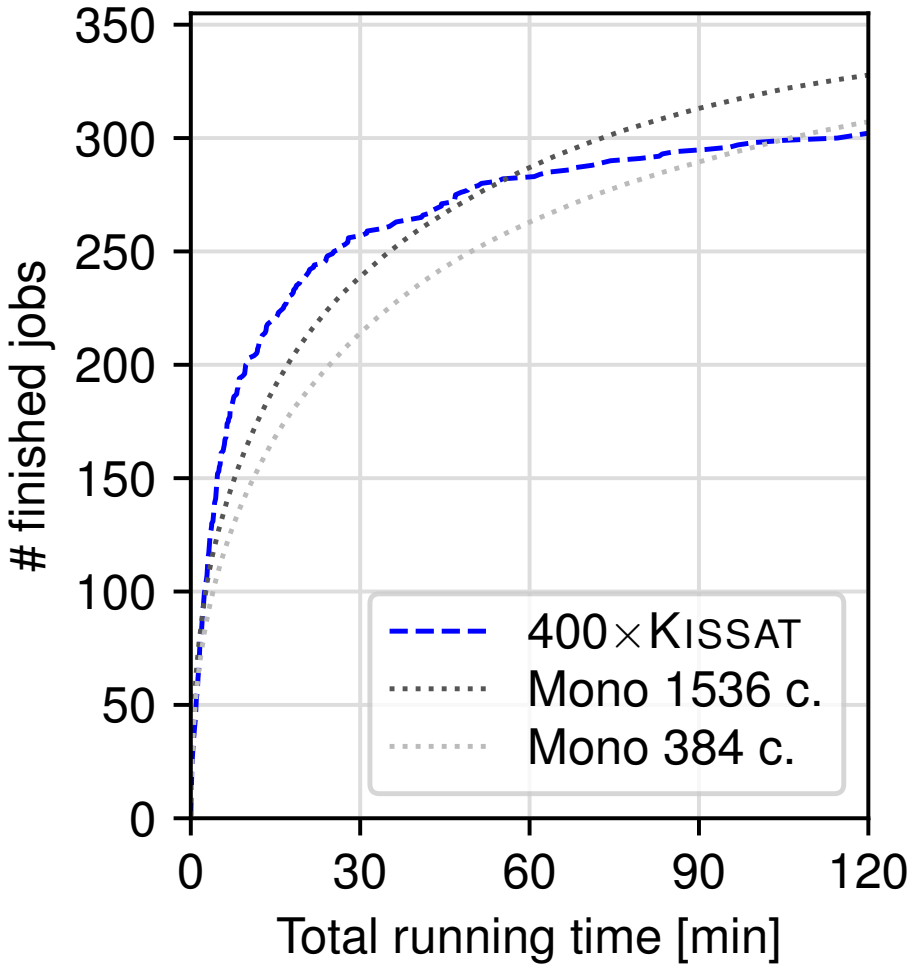
Extreme 1: 400 sequential solvers (400× Kissat)

Extreme 2: "Monolithic" parallel solving

- One job at a time
- Generous assumption: instances sorted by run time
- Maximizes per-instance speedups
- No task parallelism, poor efficiency



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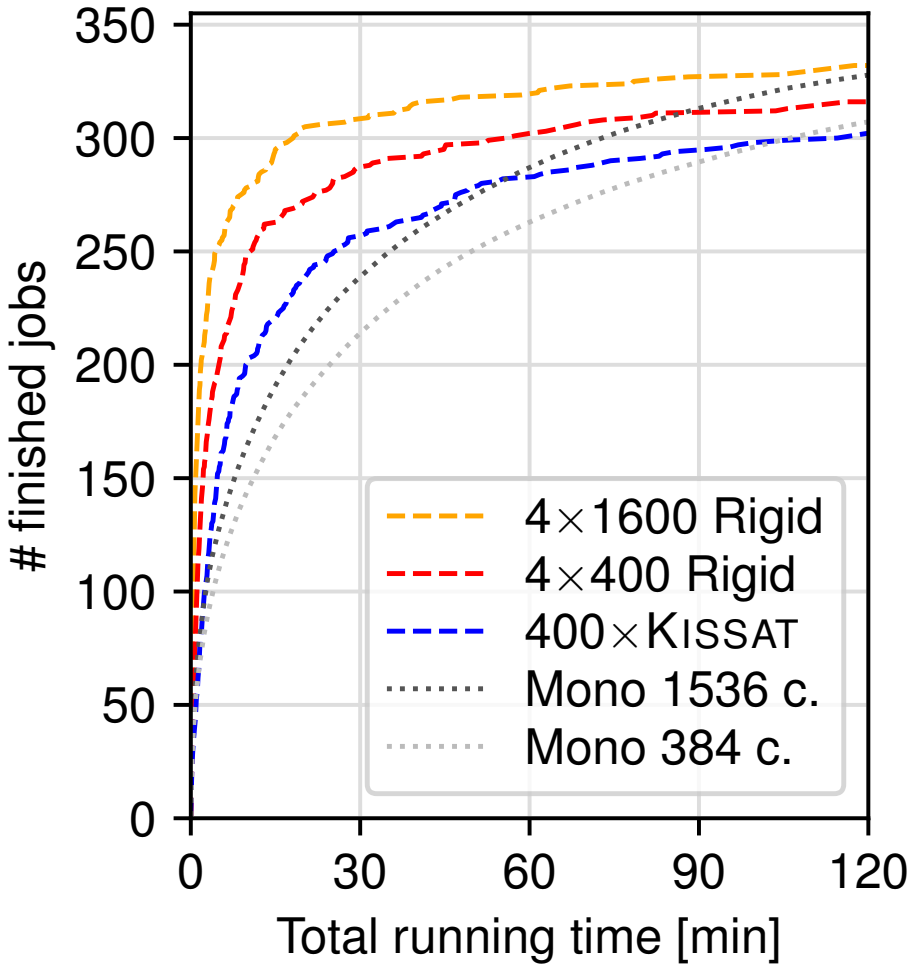
Middle ground 1: Divide cores evenly among jobs

- Solid speedups at small-scale parallel SAT
- After 15 min, $>$ 50% of cores are idling



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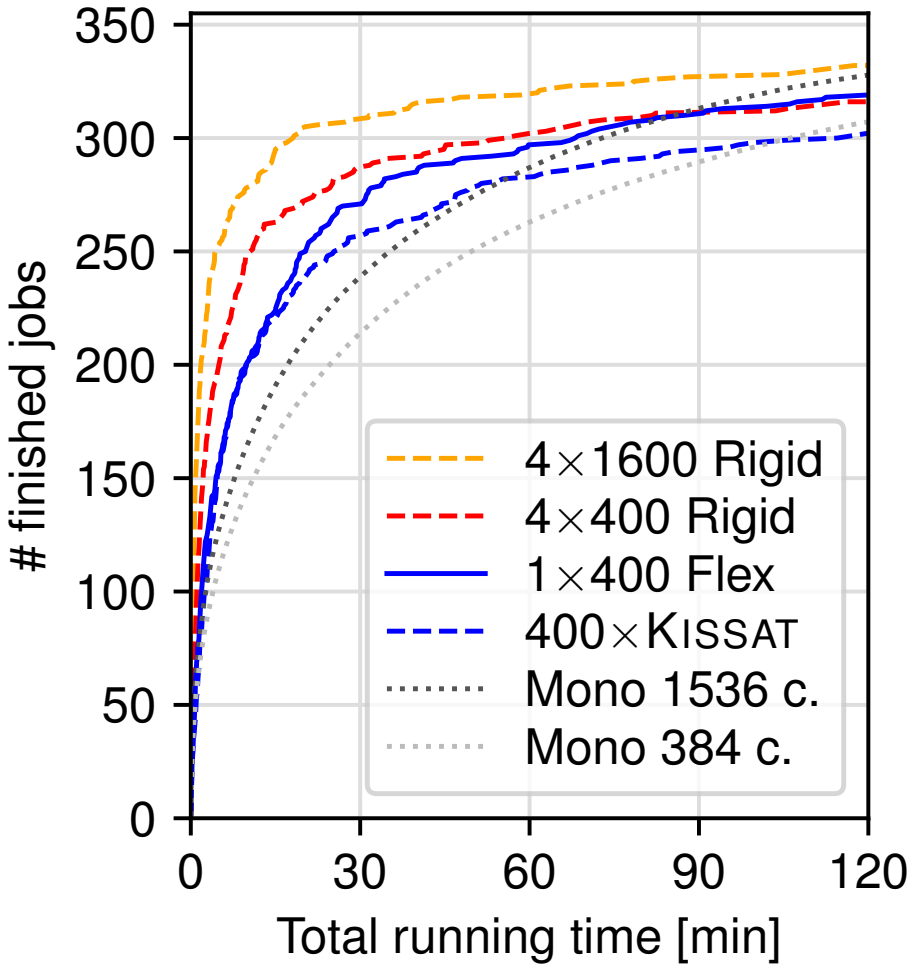

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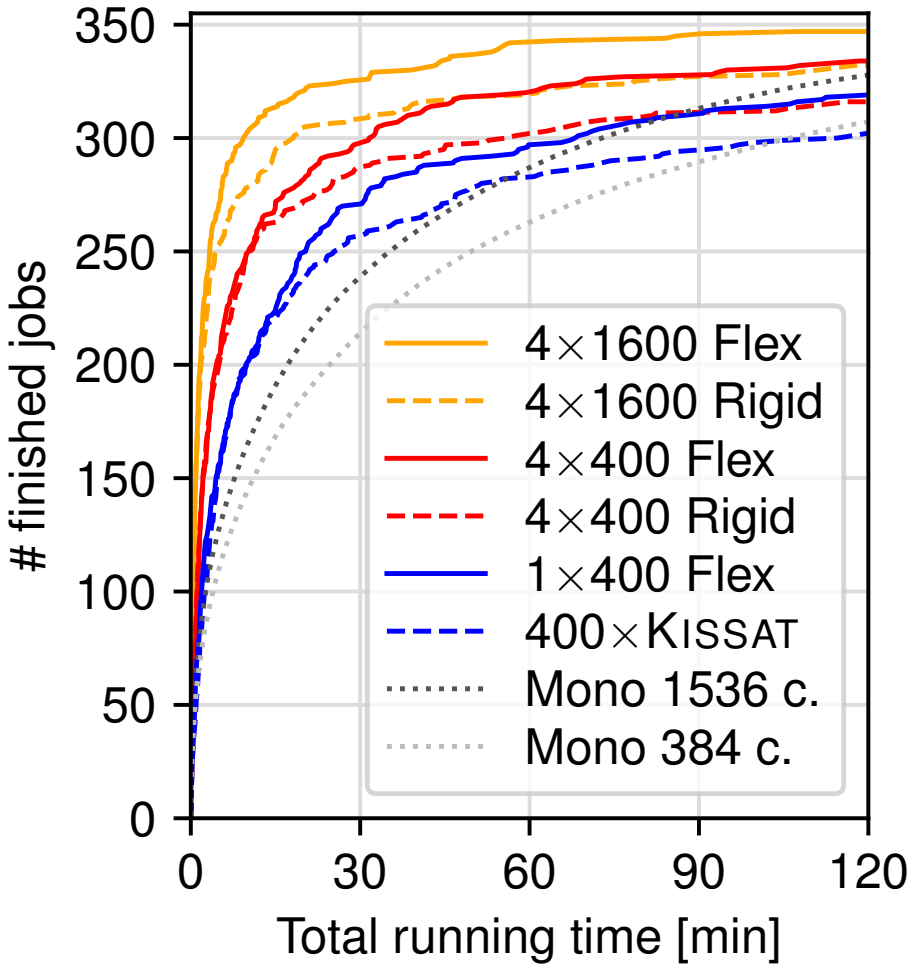
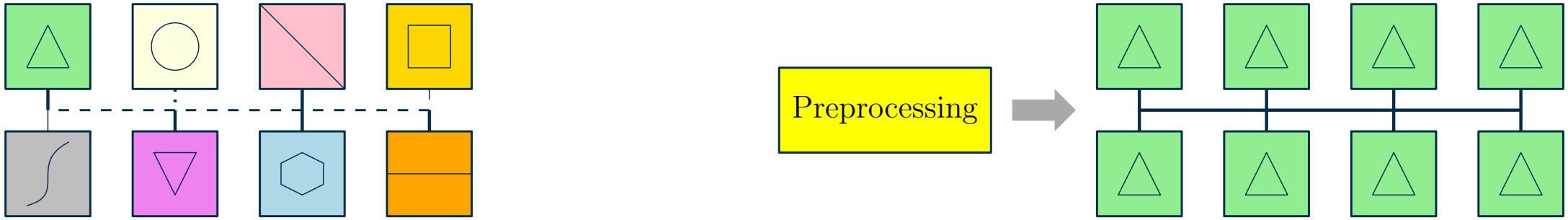
- Record number of solved instances using relatively little resources



Current Frontiers

- Streamlined clause-sharing solver design, away from portfolios [22]



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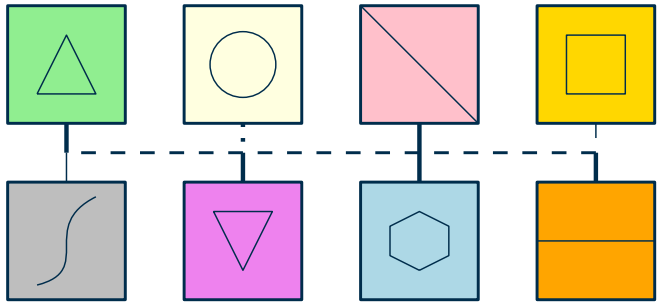

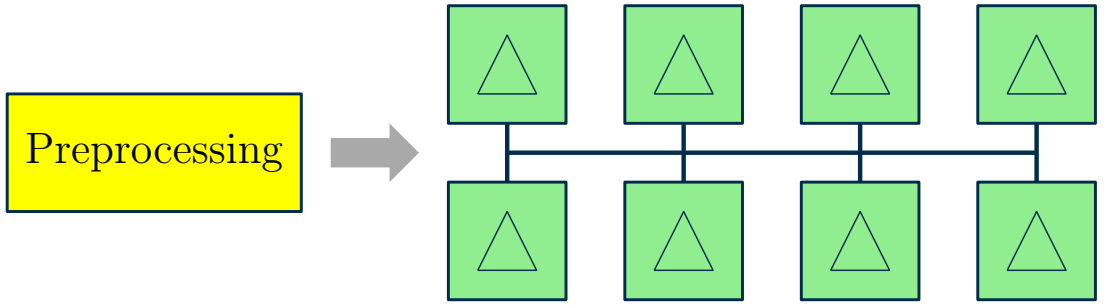

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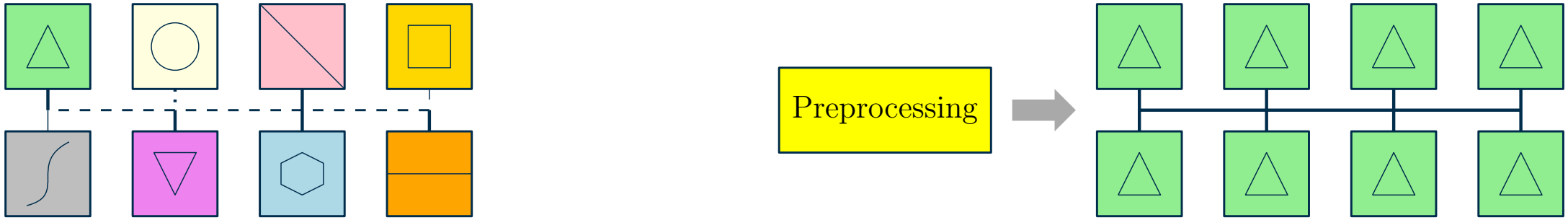

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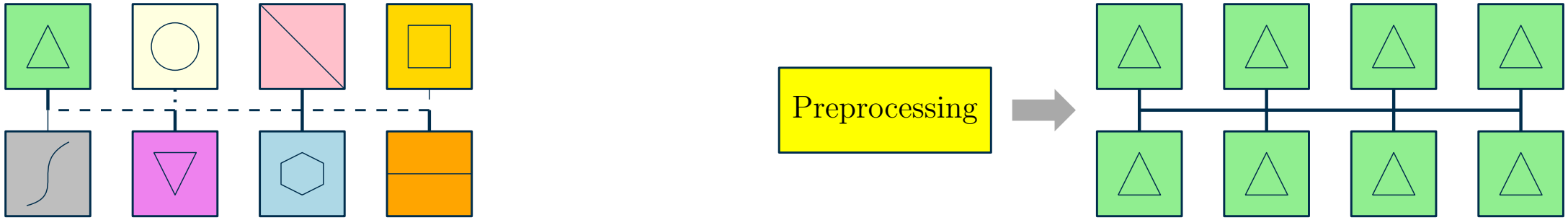

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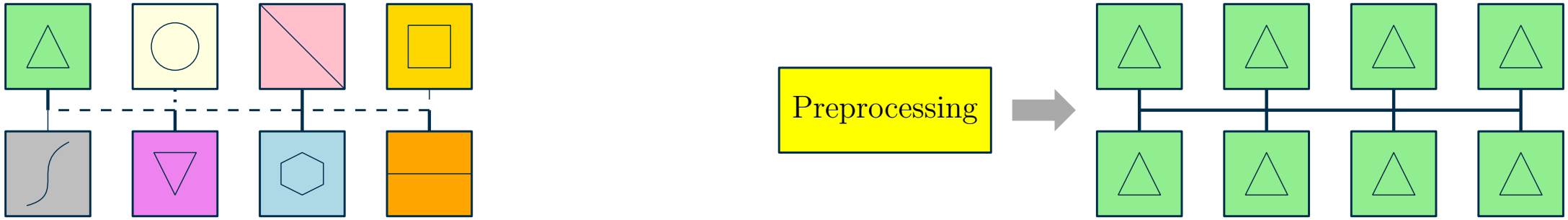

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- Proofs of unsatisfiability (next lecture!)

Recap

Parallel SAT Solving

- Popular parallelization approaches for SAT (“antisocial nerds” analogy)
  — Search space splitting, Cube&Conquer
  — Pure portfolio
  — Clause sharing portfolio
- All-to-all clause sharing can be very useful and scalable (up and down) if implemented well
  — diversifies solvers effectively in and of itself
- Exploit embarrassingly parallel job processing for interactive solving & best efficiency

Next Up

- Proof pragmatics: Formats for different proof systems in the wild
- Bringing proof technology to parallel and distributed SAT solving

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