

## 1 Recognizing Hidden Horn (6 Points)

Let  $F$  be an arbitrary CNF formula. Let  $R_F$  be a 2-SAT formula that contains the clause  $\{l_1, l_2\}$  iff there is a clause  $C \in F$  such that  $\{l_1, l_2\} \subseteq C$ . Prove that if  $R_F$  is satisfiable, then  $F$  is a Hidden Horn formula.

## 2 Tseitin Encoding (6 Points)

Encode the following formula into CNF.

- Tseitin encoding (3 Points)
- Plaisted-Greenbaum encoding (3 Points)

$$\left(\overline{x_1} \wedge \overline{(x_3 \Leftrightarrow x_2)}\right) \vee \left((x_3 \rightarrow \overline{x_4}) \wedge (x_1 \rightarrow (x_2 \wedge \overline{x_3})) \wedge x_4\right)$$

## 3 Resolution (3 Points)

Construct a resolution refutation (i.e., proof of unsatisfiability) for the following formula.

$$\{\{x_3, x_4, \overline{x_1}, x_5\}, \{\overline{x_3}, x_4, x_5\}, \{x_3, \overline{x_4}, \overline{x_1}\}, \{x_1, x_2\}, \{x_1, \overline{x_2}\}, \{\overline{x_1}, \overline{x_5}\}, \{\overline{x_3}, \overline{x_4}, x_5\}\}$$

We suggest to use this shorthand notation:  $34\overline{1}5$ ,  $\overline{3}45$ , etc.

## 4 Pythagorean Triples (3 Points)

Find a 2-coloring of the integers  $1 \leq i \leq 1000$  such that no Pythagorean triple is monochromatic.

## 5 Competition: Graph Coloring (7(+7) Points)

Implement a SAT-based graph vertex coloring solver. Your application should take as a single command line argument a DIMACS file with a graph. It needs to find the smallest number of colors needed to color the graph such that no adjacent vertices share the same color. The application should output the number of colors required.

Together with your code (and build instructions), submit a single (PDF) slide to present your solution in a two-minute lightning talk. The overall most convincing submission(s) get(s) up to **seven bonus points**.

### Code skeleton:

<https://github.com/satlecture/knit2026/blob/main/code/src/graphcol.cc>

### Benchmark instances:

<https://github.com/satlecture/knit2026/tree/main/exercises/coloring>