

1 Correctness of RAT (4 Points)

Let F be a CNF formula and c be a RAT clause for F w.r.t. some literal $x \in c$.

Show that $F \cup \{c\}$ is satisfiability-equivalent to F , i.e., $F \cup \{c\}$ is satisfiable if and only if F is satisfiable.

2 LRUP Algorithms (3 + 3 + 3 Points)

- (a) Extend the conflict analysis of CDCL to efficiently produce LRUP dependency information (“hints”).
- (b) Given a CNF formula F and an LRUP proof of unsatisfiability P for F , devise an algorithm using P that finds an *unsatisfiable core* of F , i.e., a subset of clauses in F that is unsatisfiable. The algorithm can perform a single, linear pass over P (in whichever direction) and cannot keep the entire proof in memory.
- (c) Consider a CNF formula F and an LRUP proof of unsatisfiability P produced by a parallel clause-sharing solver as in the work by Michaelson et al. (2023, 2025). Devise an algorithm that takes F and P and outputs a proof P' for F with (typically) fewer clause additions.

Hint: Parallel CDCL solver threads may produce similar sets of clauses, especially at the beginning.

3 Proof Production with Dynamic Search Space Partitioning (5 Points)

Consider a search space partitioning parallel SAT solver that dynamically splits the formula at hand. Initially, a single task is present, which processes the original input formula F . At any point, any active task processing a formula T can be replaced with two tasks $T|_x$ and $T|_{\neg x}$ (for some variable x), which then run in parallel. Each individual task uses a single sequential SAT solver that supports proof logging (e.g., via LRUP). The tasks run independently, i.e., no clause sharing takes place.

Devise an algorithm that exploits the sequential solvers’ proof logging to assemble a single, coherent proof of unsatisfiability from an UNSAT run of the described search space splitting solver.

4 Using SMT Solvers (4 Points)

Use an SMT solver to show that $x + 1 > x$ is **not** a tautology, and retrieve an according counterexample.

Prepare to demonstrate and reproduce this result live.

5 Exam Questions (0 Points)

Examine the lecture material and prepare any concrete questions you may still have.